Pooling mortality and (long-term) care risks: Life-Care Tontines

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The New York Times

THE NEW OLD AGE

Many Americans Will Need Long-Term Care. Most Won't be Able to Afford It.

A decade from now, most middle-income seniors will not be able to pay the rising costs of independent or assisted living.

Search jobs Sign in Search The Guardian International edition

Residential care costs 'can soak up over 50% of property values'

Study finds the cost of a typical residential care home stay around the UK to range from 18% to 56% of average house values

Frankfurter Allgemeine

ZEITUNG FAZ.NET

VERBRAUCHERSCHÜTZER WARNEN

„Steigende Pflegekosten sind soziale Zeitbombe"

AKTUALISIERT AM 21.08.2020 - 11:31

(The Guardian 2017, FAZ 2020, NYT 2018)

Rise in long-term care expenditure

- **Medicare program**, aiming to support **US** residents with low income in long-term care, raised from **$225 billion in 2000** (2.2% of the gross domestic product (GDP)) to **$750 billion in 2018** (3.6% of GDP).
- Belgium: LTC spending (in terms of GDP) increased from **1.7% in 2000** to **2.3% in 2018** (source: Eurostat).
- United Nations projections: The number of **elderly people**, i.e. older than 65, is projected **to triple from 2020 to 2080** to reach 2.2 billion. The global **share of the elderly population** is expected to **rise from 9.4% in 2020 to 20.6% in 2080**.

Why pool mortality and long-term care (=morbidity) risks?

- People moving into dependency need more money but have a reduced life expectancy!

  $\Longrightarrow$ **Natural hedge, diversification!**

- Individuals in bad health cannot receive long-term care insurance!

  $\Longrightarrow$ **Combined product gives access to insurance for a larger share of the population!**

- Cost reduction due to **reduced adverse selection**!

  $\Longrightarrow$ Combined product is **attractive for people in bad health...**

Why (not) mutual insurance?

- +: No guaranteed payment from insurance company / pension fund! Insurance company is only managing the product.
  $\Longrightarrow$ **Reduced risk for insurance provider!**

- $-$: Policyholder is left with significant risk (longevity, duration of long-term care)!
  $\Longrightarrow$ **Increased risk for policyholder!**

- +: Cost reduction as there is **no need for risk capital**! Long-term care risk charges are quite high...
  $\Longrightarrow$ More people might find insurance attractive...

Related literature

Mutual (life) insurance schemes gain popularity in academic literature:

- **(Natural) tontines**: Milevsky, Salisbury [2015, 2016], Chen, Hieber, Klein [2019], Chen, Hieber, Rach [2020], Chen, Rach, Sehner [2020], Chen, Qian, Yang [2020]. (...)

  $\Longrightarrow$ Lifetime-actuarially fair, continuous-time, easy-to-explain, closed pool.

- **Pooled annuities, P2P insurance, (tontines)**: Sabin [2010], Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019]. (...)

  $\Longrightarrow$ Always actuarially fair / fully funded, discrete-time.

We follow the second bullet!

Tontine products and mortality credits

Tontines were popular in the 17th / 18th century but gain popularity today:

- **Le Conservateur** (France).
- **The Tontine Trust**: `https://tontine.com/#About`.

Main idea of mortality credits: Survivors gain additional return based on (1) mortality risk and (2) amount invested.

(e.g. Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019])

Modular mutual insurance scheme: Our contribution

Based on Denuit [2019] (one-period scheme), we introduce a mutual insurance scheme that is:

1. Able to pool heterogeneous mortality risks (by age, health).
2. Discrete-time.
3. Actuarially fair and fully funded in each period.
4. Modular, flexible: Adding or removing policyholders does NOT change the AVERAGE payoff of pool members! (NEW)

Modularity allows to easily add policyholders fairly! We share the risk, the average payoff is unaffected by pooling!

Related literature and “modularity/flexibility”

Usually, the average payoff depends on the other pool members:

- Donnelly, Guillén, Nielsen [2014]:

Proposition 3.2. *Conditional upon survival, the expected instantaneous actuarial gains for individual i in the kth group are*

$$\mathbb{E}\left(dG_t^{k,i}\middle|\mathcal{F}_{t-},\, N_t^{k,i}=0\right)=\lambda_t^k W_{t-}^k\left(1-\frac{\lambda_t^k W_{t-}^k}{\sum_{m=1}^{M} W_{t-}^m\lambda_t^m L_{t-}^m}\right)dt, \quad (8)$$

for all $t\geq 0$ and for each $k=1,\ldots,M$.

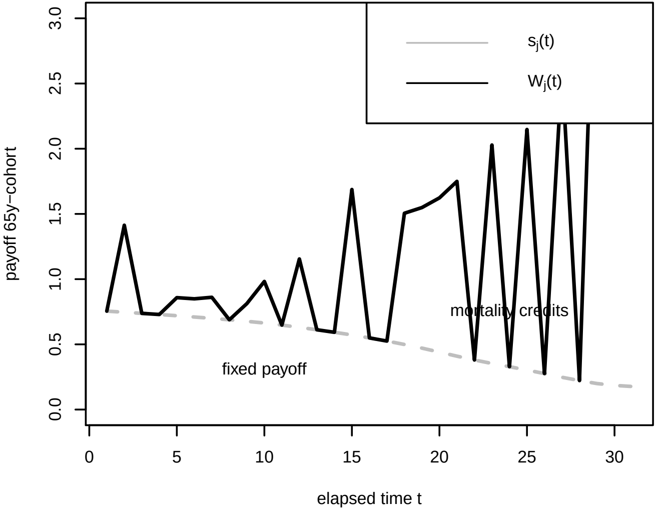
Proof. See Appendix. □

- Milevsky, Salisbury [2016]:

We use the notation E_i to remind us that this is a conditional expectation, in which $N_i-1\sim \mathrm{Bin}(n_i-1,\, {}_tp_{x_i})$, while the other $N_j\sim\mathrm{Bin}(n_j,\, {}_tp_{x_j})$. Call the above expression $w_iF_i(\pi_1,\ldots,\pi_K)$, so if $\pi=(\pi_1,\ldots,\pi_K)$, then

$$F_i(\pi)=\int_0^\infty e^{-rt}\,{}_tp_{x_i}\,wd(t)\,E_i\left[\frac{\pi_i}{\sum_j \pi_j w_j N_j(t)}\right]dt$$

Problem: Difficult to allow people to join the scheme later (closed-funds (Milevsky, Salisbury [2016])). Average payoffs are difficult to predict and depend on other (possibly future) pool members.


$s_j(t)$
$W_j(t)$
mortality credits
fixed payoff
payoff 65y−cohort
elapsed time t
0.0
0.5
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2.5
3.0
0
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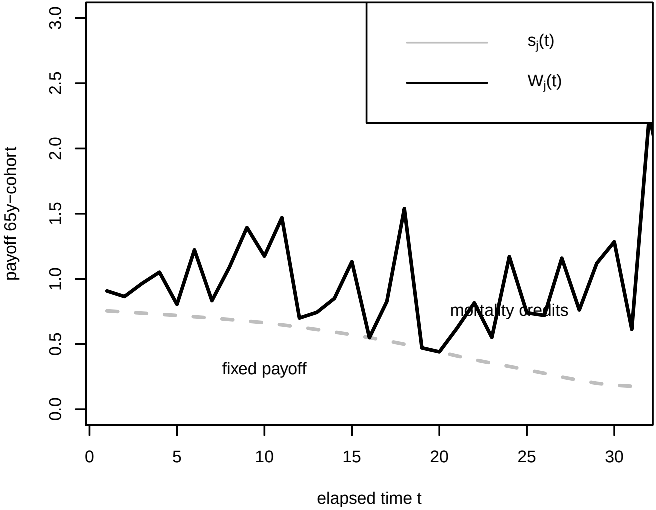
$n = 10$


$s_j(t)$
$W_j(t)$
payoff 65y−cohort
elapsed time t
fixed payoff
mortality credits
0.0
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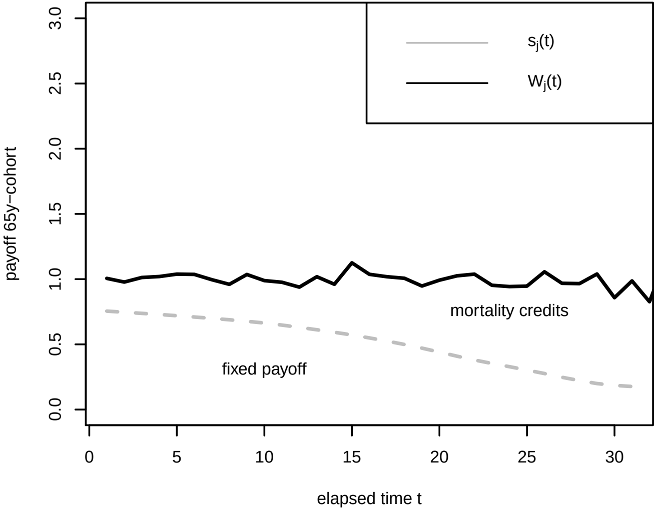
$n = 50$


$s_j(t)$
$W_j(t)$
mortality credits
fixed payoff
payoff 65y−cohort
0.0
0.5
1.0
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2.5
3.0
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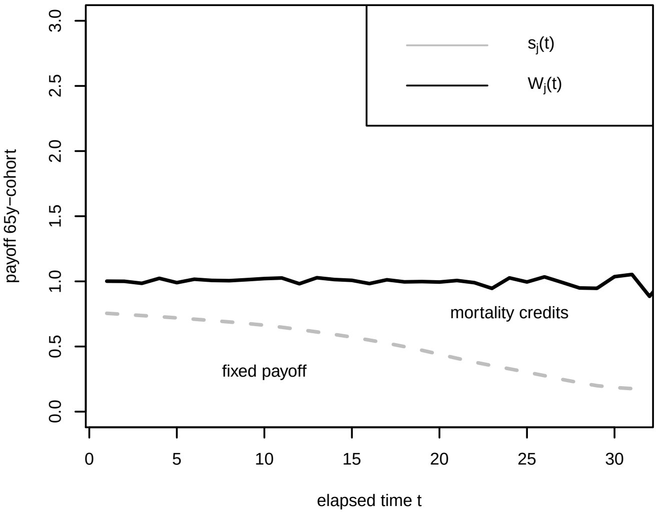
elapsed time t

$n = 1\,000$



$s_j(t)$
$W_j(t)$
mortality credits
fixed payoff
payoff 65y−cohort
elapsed time t
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
5
10
15
20
25
30

$n = 10\,000$

Some notation

- Pool members $\mathcal{L}_0 = \{1, 2, ..., n\}$. Time in periods $t = 0, 1, 2, \ldots$.
- Individual $j \in \mathcal{L}_0$ contributes single premium $c_j(0)$ at time 0.
- Deterministic, risk-free rate δ_t, $t \geq 0$.
- Remaining lifetimes $T_j, j \in \mathcal{L}_0$, are assumed to be independent.
- Survivors: $\mathcal{L}_t = \{j \in \mathcal{L}_0 \mid T_j > t\}$.
  Deceased in $(t-1, t]$: $\mathcal{D}_t = \mathcal{L}_{t-1} - \mathcal{L}_t$.
- Survival probability: ${}_tp_{x_j} = \mathbb{E}[\mathbb{1}_{T_j > t}] = \mathbb{E}[\mathbb{1}_{j \in \mathcal{L}_t}], \quad p_{x_j} := {}_1p_{x_j}$.
  Death probability: $q_{x_j} := 1 - p_{x_j}$. Maximal age $\omega \in \mathbb{N}$.
- Individual account value, fixed payoff $s_j(t)$:

$$c_j(t) = \begin{cases} e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1) - s_j(t)\,, & j \in \mathcal{L}_t \\ 0\,, & \text{otherwise} \end{cases} \tag{1}$$

Mutual insurance: Insurer's view and actuarial fairness

For each $t = 0, 1, \ldots$, the premium equivalence holds: (pool view)

$$\underbrace{\sum_{j=1}^{n} c_j(t)}_{\text{total account values}} = \sum_{j=1}^{n} \underbrace{\sum_{s=t+1}^{\omega - x_j} e^{-\int_t^s \delta_u \mathrm{d}u} W_j(s)}_{\text{discounted future benefits individual } j} . \tag{2}$$

- Right hand side: **random** (big letter!)
- Left hand side: **deterministic**.

For each $t = 0, 1, \ldots$, the contract is fully-funded: (individual view)

$$\underbrace{c_j(t)}_{\text{retrospective reserve}} = \underbrace{\mathbb{E}_t\left[\sum_{s=t+1}^{\omega - x_j} e^{-\int_t^s \delta_u \mathrm{d}u} W_j(s)\right]}_{\text{prospective reserve}} . \tag{3}$$

Mortality credits

In case of death of individual j, the following is redistributed within the pool:

$$X_j(t) := \mathbb{1}_{j \in \mathcal{D}_t} \cdot e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1)\,,$$

resulting in the pool's total time-t mortality credit

$$X(t) := \sum_{j \in \mathcal{L}_{t-1}} X_j(t) = \sum_{j \in \mathcal{D}_t} e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1)\,.$$

An individual $j \in \mathcal{L}_{t-1}$ receives a payoff of:

$$W_j(t) = \begin{cases} s_j(t) + \beta_j\big(X(t)\big), & \text{if } j \in \mathcal{L}_t \\ \beta_j\big(X(t)\big), & \text{if } j \in \mathcal{D}_t \end{cases} \tag{4}$$

decomposed of

- $s_j(t)$: individual, fixed withdrawal amount,
- $\beta_j\big(X(t)\big)$: collective part of the benefits, i.e. the mortality credits.

Essential part of modularity is a (small) death benefit $\beta_j\big(X(t)\big)$!

Definition (Fair distribution rule: mortality credits)

A fair distribution rule $\beta_j\big(X(t)\big)$ satisfies:

- **Self-sufficiency property**: $\sum_{j\in\mathcal{L}_{t-1}} \beta_j\big(X(t)\big) = X(t)$.
- **Positivity property**: $\beta_j\big(X(t)\big) \geq 0$.
- **Fairness property**:

$$\mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\right] \quad = \quad \underbrace{\mathbb{E}_{t-1}\left[\mathbb{1}_{j\in\mathcal{D}_t}\right]}_{\text{probability to die in } (t-1,\, t]} \quad \cdot \underbrace{e^{\int_{t-1}^{t}\delta_s \mathrm{d}s} c_j(t-1)}_{\text{amount at risk at time } t}, \qquad (5)$$

where $\mathbb{E}_t := \mathbb{E}[\,\cdot\,|\,\mathcal{F}_t]$ is an expectation conditional on the information $\mathcal{F}_t := \sigma(\mathcal{L}_t)$.

Example (Linear risk sharing rule)

At time t, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$\beta_j\big(X(t)\big) = \frac{q_{x_j+t-1} \cdot c_j(t-1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j+t-1} \cdot c_j(t-1)} \cdot X(t)\,. \tag{6}$$

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018]

Example (Conditional mean risk sharing rule)

At time t, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$\beta_j\big(X(t)\big) = \mathbb{E}_{t-1}[\,X_j(t)\,|\,X(t)\,]\,. \tag{7}$$

(see, e.g., Denuit and Dhaene [2012], Denuit [2019])

Individual $j \in \mathcal{L}_t$'s time-t account value is given by:

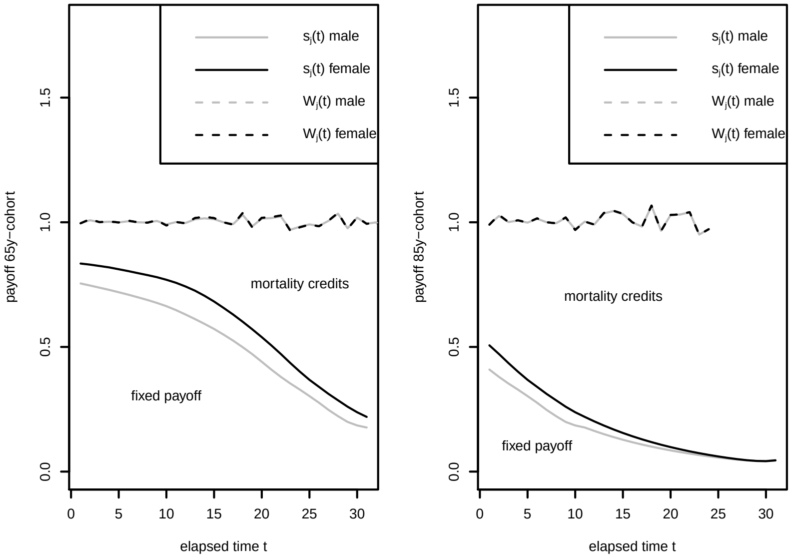
$$c_j(t) = \sum_{u=t+1}^{\omega - x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u)\,. \tag{8}$$

How do we choose $s_j(u)$, $u = 1, 2, \ldots, \omega - x_j$?

For example, choose the average payoff to be constant, equal to $b_j > 0$:

$$\begin{aligned} \mathbb{E}_{t-1}[W_j(t) \,|\, j \in \mathcal{L}_t] &= \mathbb{E}_{t-1}\left[\mathbb{1}_{j \in \mathcal{L}_t} \cdot s_j(t) + \mathbb{1}_{j \in \mathcal{L}_{t-1}} \cdot \beta_j\big(X(t)\big) \,|\, j \in \mathcal{L}_t\,\right] \\ &= s_j(t) + \mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\right] \\ &= s_j(t) + q_{x_j+t-1} e^{\int_{t-1}^t \delta_s \mathrm{d}s} c_j(t-1) \stackrel{!}{=} b_j\,. \end{aligned} \tag{9}$$

((9) is a system of equations backwards in time!)



$s_j(t)$ male
$s_j(t)$ female
$W_j(t)$ male
$W_j(t)$ female
payoff 65y−cohort
mortality credits
fixed payoff
elapsed time t
0.0
0.5
1.0
1.5
0
5
10
15
20
25
30
payoff 85y−cohort

Theorem (Backwards iteration)

If an individual $j \in \mathcal{L}_t$ aims for an average payoff $b_j(t)$, the fixed payoff is given by:

$$s_j(t) = \begin{cases} \frac{b_j(t)}{1+q_{\omega-1}}, & \text{for } t = \omega - x_j \\ \frac{b_j(t) - q_{x_j+t-1} \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u)}{1+q_{x_j+t-1}}, & \text{for } t = \omega - x_j - 1, \omega - x_j - 2, \ldots, 1 \end{cases} \tag{10}$$

We derive the individual's account value as

$$c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u) \tag{11}$$

and the initial single premium as $c_j(0)$.

Discussion

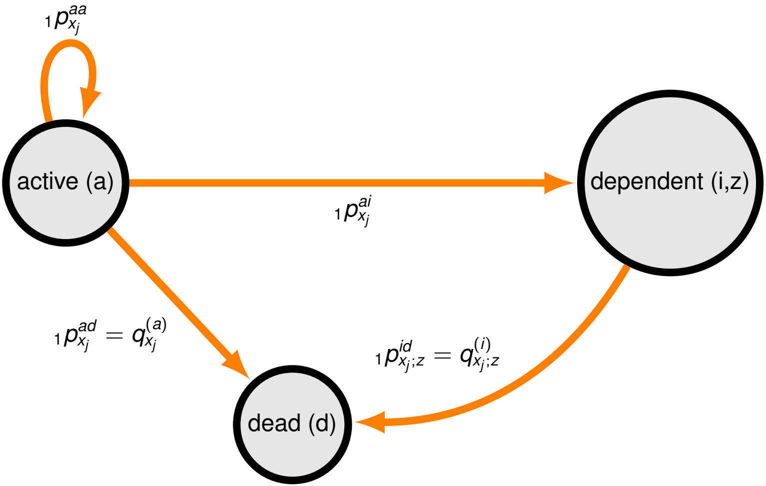
- The backwards iteration detects the split between **fixed payoff** $s_j(t)$ and **mortality credits** $\beta_j\big(X(t)\big)$ that leads to an average payoff of $b_j(t)$.
- The backwards iteration can be carried out **individually** for each $j \in \mathcal{L}_0$ (modularity / flexibility).
- This allows **different age cohorts** to share mortality risks in a fair way.

Motivation

A fair, heterogeneous, modular mutual insurance scheme

The Life-Care Tontine

Life-Care Tontine: semi-Markov model



z: time spent in dependency.

Life-Care Tontine: Mortality credits

In a first step, we **adapt payments in dependency**:

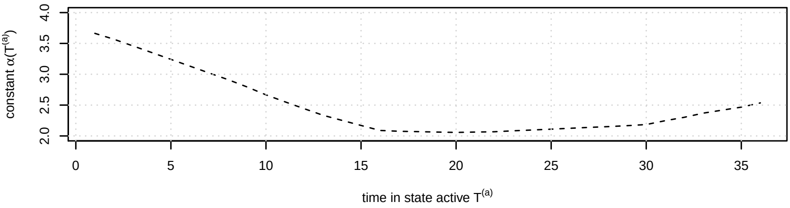
$$\mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\,\middle|\, j \in \mathcal{A}_{t-1}\right] = q^{(a)}_{x_j+t-1} \cdot e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j^{(a)}(t-1)\,, \tag{12}$$

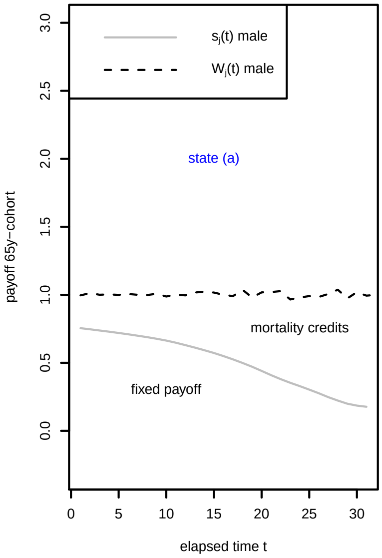
$$\mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\,\middle|\, j \in \mathcal{I}_{t-1;z}\right] = q^{(i)}_{x_j+t-1;z} \cdot e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j^{(i)}(t-1;z)\,. \tag{13}$$

Adjusting benefits in dependency

A dependent individual receives an average **increased payment** $\alpha_j(T^{(a)}) \cdot b_j(t)$ in dependency with $\alpha_j(T^{(a)}) > 1$ and $T^{(a)}$ the time spent in active state.

Fair adjustment constant based on French long-term care (LTC) data:





payoff 65y−cohort
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$s_j(t)$ male
$W_j(t)$ male
state (a)
mortality credits
fixed payoff
0
5
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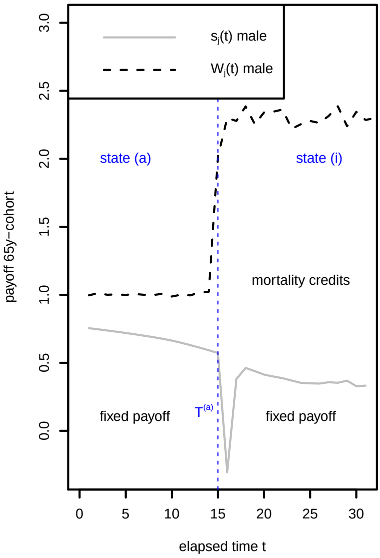
elapsed time t



payoff 65y−cohort
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$s_j(t)$ male
$W_j(t)$ male
state (a)
state (i)
mortality credits
fixed payoff
$T^{(a)}$
fixed payoff
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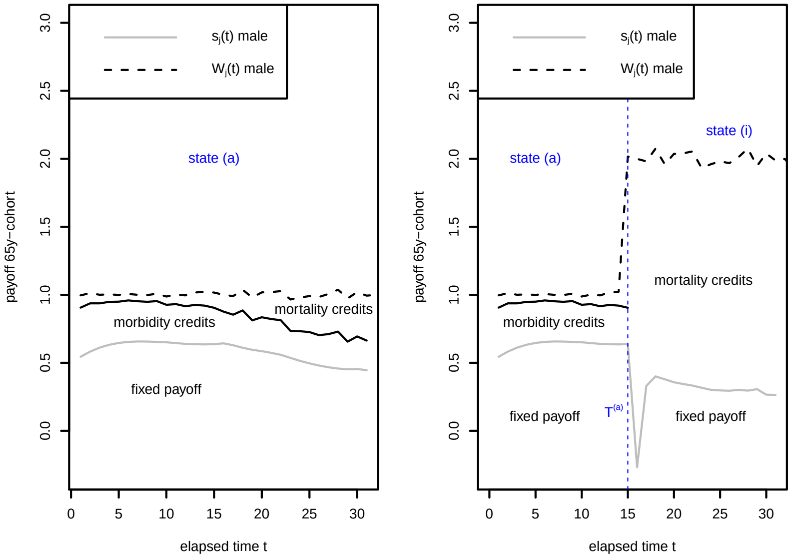
elapsed time t



$s_j(t)$ male
$W_j(t)$ male
state (a)
morbidity credits
mortality credits
fixed payoff
payoff 65y−cohort
elapsed time t
0
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0.0
0.5
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1.5
2.0
2.5
3.0
$s_j(t)$ male
$W_j(t)$ male
state (a)
state (i)
mortality credits
morbidity credits
fixed payoff
$T^{(a)}$
fixed payoff
payoff 65y−cohort
elapsed time t

Discussion and conclusion

- It is beneficial to **pool mortality and long-term care (morbidity) risks**.
- We propose a fair, modular / flexible mutual insurance scheme ($b_j(t)$ for each individual j, we share the risk, the average payment is unaffected by pooling!).
- We show how this scheme can be adapted to a **life-care tontine** introducing the concept of **morbidity credits**.
- The scheme allows to pool different age cohorts.
- It is **fully-funded at all times**, allowing individuals to later join the scheme!

Thank you!

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