Institut für Finanz- und Aktuarwissenschaften ifa

Pricing longevity-linked securities in the presence of mortality trend changes

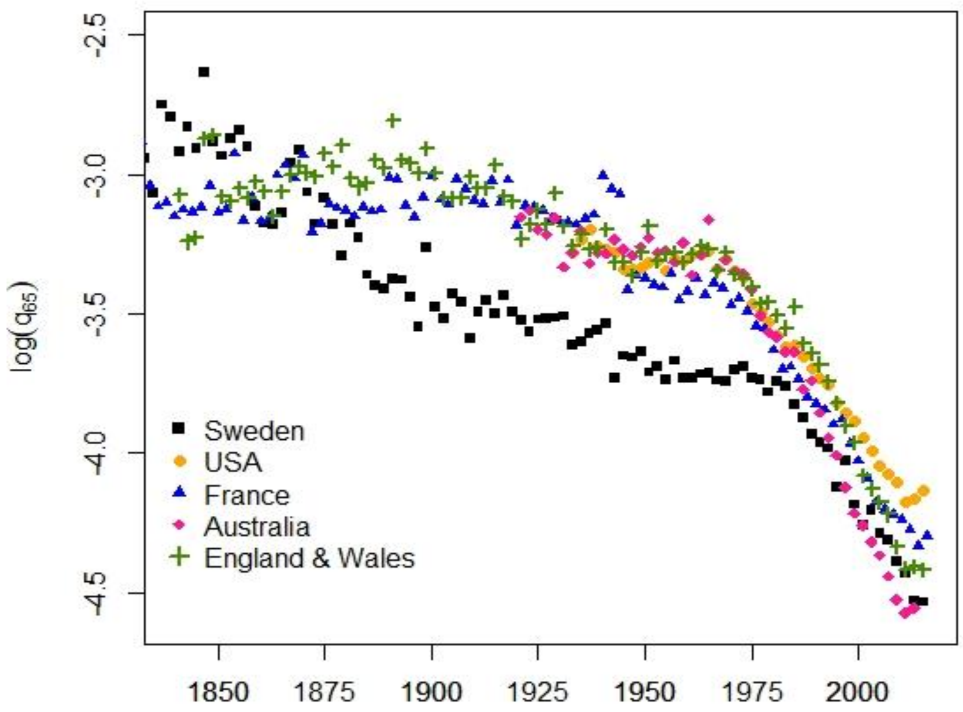
- Arne Freimann
- Working Paper
- Research Workshop IVW – ifa
- February 2020

Pricing longevity-linked securities in the presence of mortality trend changes

Agenda

Introduction

Trend changes in historical mortality patterns



- **Structural changes** in the long-term mortality trend are observable for many countries (Li et al. (2011), Börger & Schupp (2018))
- Major risk for entities that are exposed to **longevity risk**

Introduction
Longevity-linked securities

- **Longevity risk transfer market („new life market")**
  - Emerged in the UK in 2006
  - Academics have proposed several instruments to transfer longevity risk (Blake et. al (2018))
  - Huge success of insurance-based „customized longevity swaps"
  - Market is still illiquid and incomplete
  - Potential size of the global longevity risk market for pension liabilities between $60trn and $80trn (Michaelson and Mulholland (2015))
- **Stochastic mortality modeling**
  - Required for modeling, quantification, and management of longevity risk
  - Typically, these models capture „diffusion risk" around a constant trend (random walk with drift)
  - The possibility of future mortality trend changes is often left unmodeled (Sweeting (2011), Liu & Li (2017), Börger & Schupp (2018))

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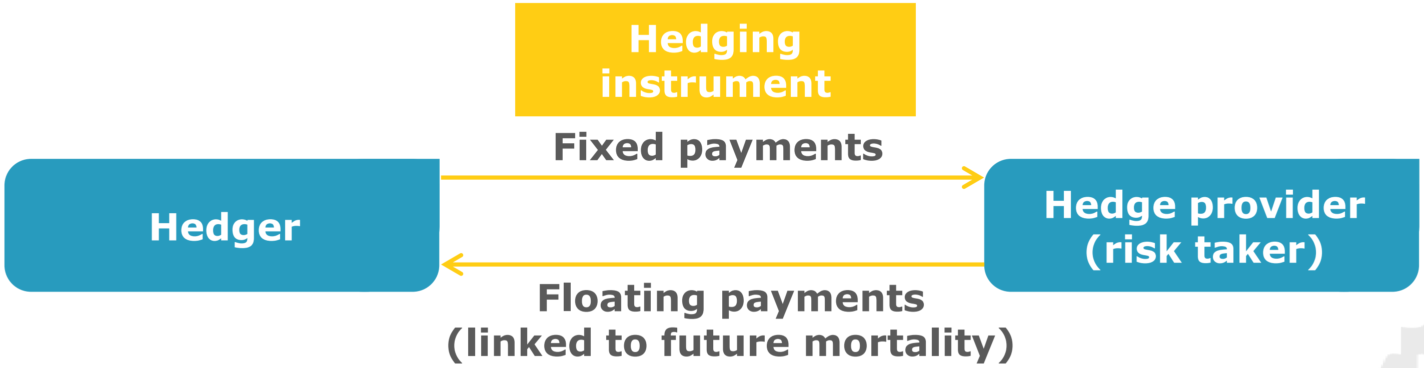
Introduction
Pricing

- **Promising approaches**
  - **Risk-adjusted (risk-neutral) approach**
    - Change of measure inspired by capital market theory (Cairns et. al (2006))
    - Prices are derived as expected values of discounted future cash flows
  - **Cost of capital approach**
    - Inspired by regulatory capital requirements for reinsurers (Börger (2010))
    - Expected return should exceed the additional capital charges for taking the risk
    - High practical relevance (Levantesi and Menzietti (2017), Zeddouk and Devolder (2019))
- **Shortcoming**
  - Practical suitability highly depends on the adequacy of the underlying stochastic mortality model
  - Ignoring the risk of mortality trend changes might significantly underestimate the risks taken

Objective
Implement and apply both approaches in a framework which explicitly allows for random future changes in the long term mortality trend

Modeling framwork



- Longevity risk exposure
  - Simplified Portfolio of (deferred) annuities
  - Book population is subpopulation of larger reference population *R*
  - Specific socioeconomic structure
  - Closed to new business

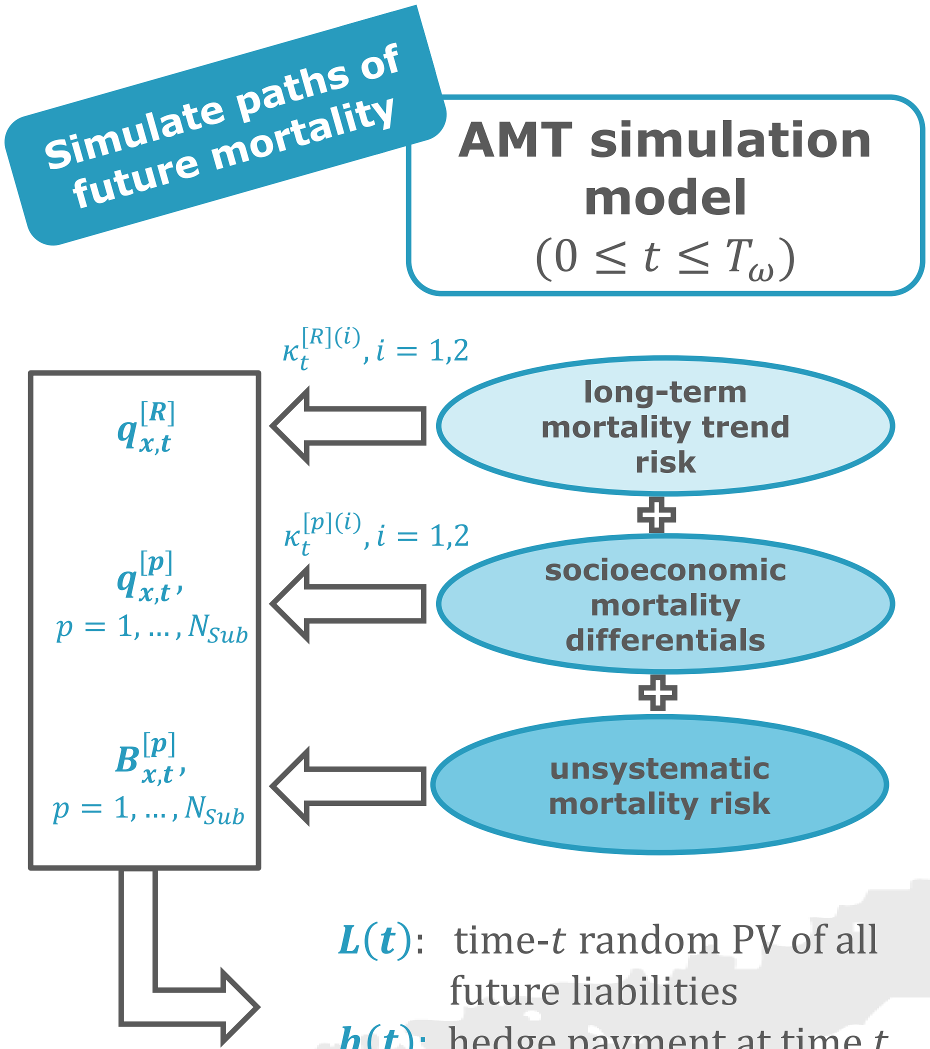
- Appetite for taking longevity risk
  - Exploit diversification benefits
  - Earn a risk premium
  - No counterparty credit risk

Focus on the hedge provider

- How much risk is taken?
- For which price is he willing to assume this risk?

Modeling framework

Multi-population AMT simulation model



$L(t)$: time-t random PV of all future liabilities

$h(t)$: hedge payment at time t

$H(t)$: time-t random PV of all future hedge payments

CBD model structure (Cairns et al. (2006))

- Reference population *R*

$$logit\left(q_{x,t}^{[R]}\right) := log\left(\frac{q_{x,t}^{[R]}}{1 - q_{x,t}^{[R]}}\right) = k_t^{(1)[R]} + (x - \bar{x})k_t^{(2)[R]}$$

- Socioeconomic subpopulations $p = 1, \dots, N_{Sub}$

$$logit\left(q_{x,t}^{[p]}\right) - logit\left(q_{x,t}^{[R]}\right) = k_t^{(1)[p]} + (x - \bar{x})k_t^{(2)[p]}$$

Stochastic trend process (Börger & Schupp (2018))

- $k_t^{(i)[R]} = \bar{k}_t^{(i)[R]} + \varepsilon_t^{(i)[R]}, \quad i = 1,2$
  - **Random noise** around piecewise linear trend
- $\bar{k}_{t+1}^{(i)[R]} = \bar{k}_t^{(i)[R]} + AMT_t^{(i)}, \quad i = 1,2$
  - Actual mortality trend $AMT_t^{(i)} = AMT_{t-1}^{(i)} + O_t^{(i)} S_t^{(i)} M_t^{(i)}$
    - $O_t^{(i)} \in \{0,1\}$ did a trend change **occur**?
    - $S_t^{(i)} \in \{-1,1\}$ **sign** of trend change
    - $M_t^{(i)} > 0$ absolute trend change **magnitude**

Random walk with drift / autoregressive process

- $\varepsilon_t^{(i)[p]}$ annual random innovations for subpopulation p

Sampling survivors from a Binomial distribution

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Pricing approaches
Risk-adjusted measure

- **Risk-adjusted version of the AMT simulation model**
  - Adjust the distribution of each individual risk driver
  - Technique of multivariate normalized exponential tilting (Wang (2007), Chen and Cox (2009))

Risk driver	**Objective dynamics** ($\mathbb{P}$)	**Risk-adjusted dynamics** ($\mathbb{Q}$)
Trend change occurence	$Bernoulli(p^{(i)})$	$Bernoulli\left(\Phi\left(\Phi^{-1}(p^{(i)}) + \boldsymbol{\lambda_O^{(i)}}\right)\right)$
Trend change sign	$\mathbb{P}\left(S_t^{(i)} = -1\right) = 0.5$	$\mathbb{Q}\left(S_t^{(i)} = -1\right) = \Phi\left(\Phi^{-1}(0.5) + \boldsymbol{\lambda_S^{(i)}}\right)$
Trend change magnitude	$LN\left(\mu_M^{(i)}, {\sigma_M^{(i)}}^2\right)$	$LN\left(\mu_M^{(i)} + \boldsymbol{\lambda_M^{(i)}}\sigma_M^{(i)}, {\sigma_M^{(i)}}^2\right)$
Noise around AMT	$N\left(0, \Sigma^{[R]}\right)$	$N\left(0 - \boldsymbol{\Lambda_\varepsilon}, \Sigma^{[R]}\right)$
Socioec. mortality differentials	$N\left(0, \Sigma^{[Sub]}\right)$	$N\left(0 - \boldsymbol{\Lambda_{Sub}}, \Sigma^{[Sub]}\right)$

- Hedge premium $P_0 := \mathbb{E}^{\mathbb{Q}}(H(0)) = \underbrace{\mathbb{E}^{\mathbb{P}}(H(0))}_{\text{Best estimate}} + \underbrace{\left(\mathbb{E}^{\mathbb{Q}}(H(0)) - \mathbb{E}^{\mathbb{P}}(H(0))\right)}_{\text{Risk loading}}$

Market price of risk $\lambda_{Risk}^{(i)} > 0, i = 1,2$ for the individual longevity risk drivers $Risk \in \{O, S, M, \varepsilon, Sub\}$ need to be specified

Pricing approaches

Cost of capital approach: Theory

- **Solvency Capital Requriements (SCRs)**
  - Back business with sufficient economic capital
  - Under Solvency II, the SCR for issuing a longevity-linked security H corresponds to the 99.5% quantile of (cf. Börger (2010)):

$$\frac{\widetilde{H}(T+1) + h(T+1)}{1+r} - \widetilde{H}(T)$$

    - Potential loss if next year's **realized mortality** will be lower than anticipated
    - Potential loss due to **revised long-term mortality assumptions** for the time beyond
- **Cost of capital**

$$CoC_H = \sum_{t\geq 0} \frac{r_{CoC} SCR_H(t)}{(1+r)^{t+1}}$$

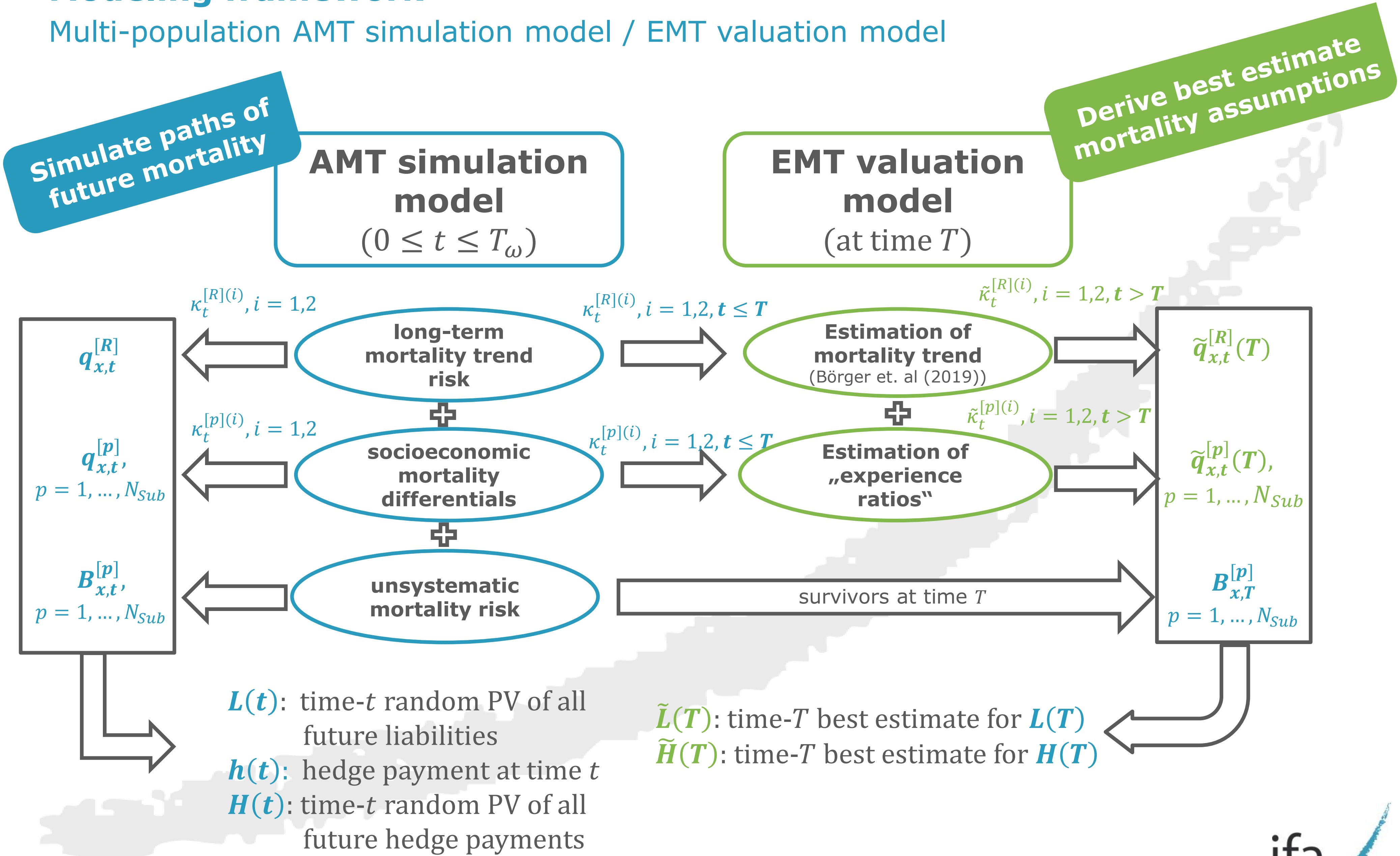
  - r_{CoC} cost of capital rate: minimum rate of return that shareholders demand for providing capital

Ignoring diversification benefits, the hedge provider will be willing to offer the instrument at the objective best estimate value plus the expected cost of capital

$$P_0 := \underbrace{\mathbb{E}^{\mathbb{P}}\big(H(0)\big)}_{\text{Best estimate}} + \underbrace{\mathbb{E}^{\mathbb{P}}(CoC_H)}_{\text{Risk loading}}$$

Modeling framework

Multi-population AMT simulation model / EMT valuation model

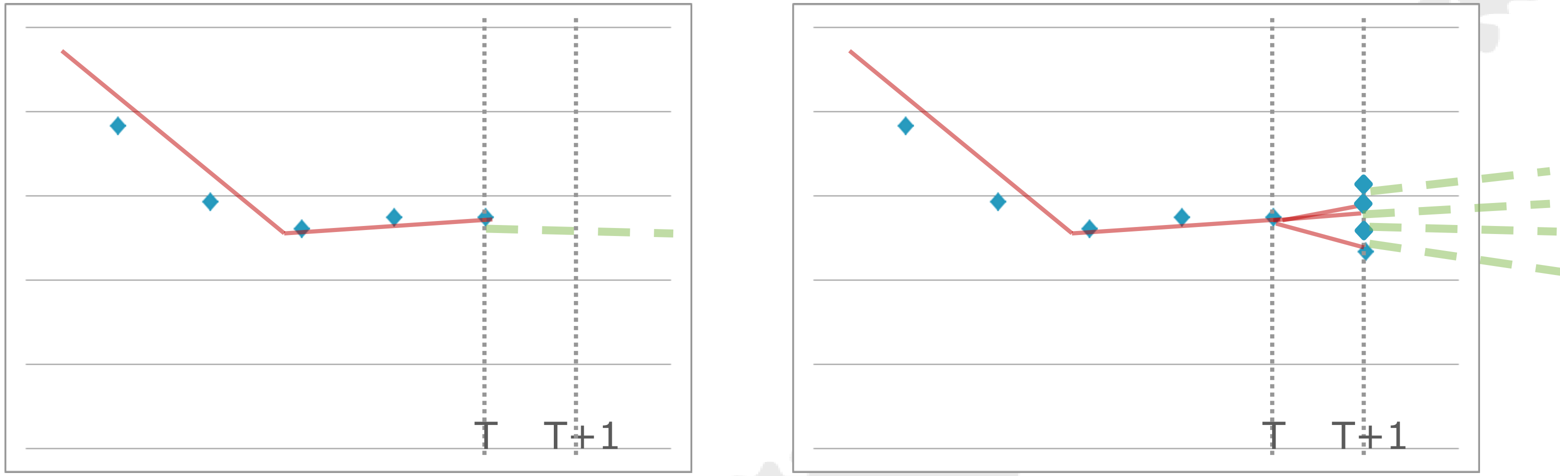


Pricing approaches

Cost of capital approach: Implementation

Illustration: one outer model path:

$$\frac{\widetilde{H}(T+1) + h(T+1)}{1+r} - \widetilde{H}(T)$$



In a two-level nested simulation, **entire distributions** can be derived for

- the required economic capital $SCR_H(T)$ and
- the cost of capital CoC_H

Pricing approaches

Discussion and qualitative comparison

Risk-adjusted measure		Cost of capital approach
• Not fully justified in an incomplete market	**Economic justification**	• Of high practical relevance • Only applies to reinsurers
• Model structure is preserved • Applicable to any instrument • Pricing operator is linear	**Practical applicability**	• In the presented form, limited to symmetric payout structures • Pricing operator is not linear
• Market price $\lambda_{Risk}^{(i)}$ for each longevity risk driver • Restriction: $\lambda_{Risk}^{(i)} \equiv \lambda$	**Calibration**	• Single pricing parameter r_{CoC} • Starting point $r_{CoC} = 6\%$

To have fully specified pricing models at hand, r_{CoC} and λ need to be specified

Numerical illustrations

Overview of model parameters

- **Model calibration**
  - **Reference population**: National population of English and Welsh males (Human Mortality Database (2018))
  - **Subpopulations**: Five subpopulations of different socioeconomic status based on the Index of Multiple Deprivation (IMD) for England (Office for National Statistics (2018))
- **Choice of parameters**

Description	Parameter
Initial age of policyholders	$x_0 = \in \{50,65,80\}$
Retirement age	$x_R = 65$
Initial portfolio size	$10{,}000$
Socioeconomic book composition	$(0, 0, 0.3, 0.3, 0.4)$
Risk-free interest rate	$r = 2\%$
Cost of capital rate	$r_{CoC} = 6\%$
Market price of longevity risk drivers	λ (to be determined)

Hedging instrument

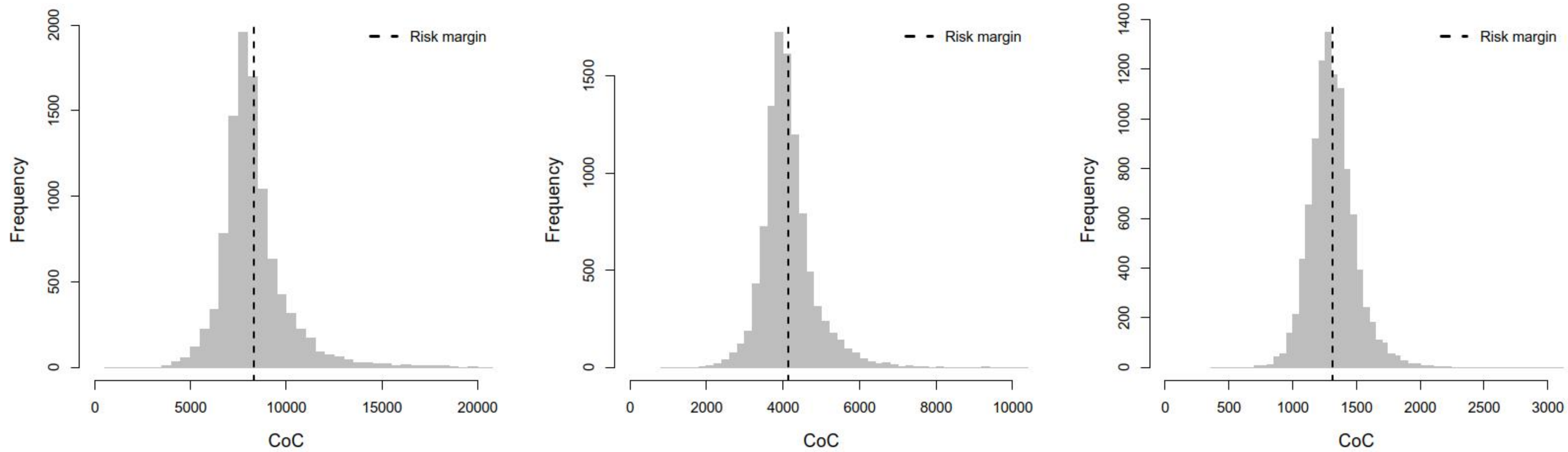
Longevity swaps

- **Payout to the hedge provider**

$$h(t) := \mathbb{1}_{\{x_0+t \geq x_R\}}\left(\boldsymbol{F}_{\boldsymbol{x_0+t,t}} - \boldsymbol{S}_{\boldsymbol{x_0+t,t}}\right), \qquad 0 \leq t \leq \tau$$

  - Maturity τ
  - $\boldsymbol{F}_{\boldsymbol{x_0+t,t}}$: receives fixed forward rates (fixed at inception)
    - Forward Rates = Best estimate + Risk loading
  - $\boldsymbol{S}_{\boldsymbol{x_0+t,t}}$: pays floating number of survivors (realized at time t)
    - **Customized**
      - Linked to actual number of survivors in the book portfolio
      - Unlimited $(\tau = T_\omega)$ design provides a perfect hedge
    - **Index-based**
      - Linked to ex-post survival probability in the reference population
      - Hedger: population basis risk

The risk margin and the market price of longevity risk

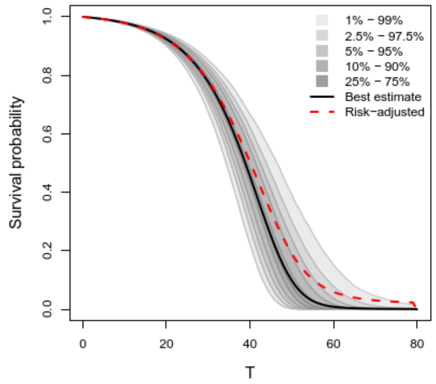


(a) $x_0 = 50$ (b) $x_0 = 65$ (c) $x_0 = 80$

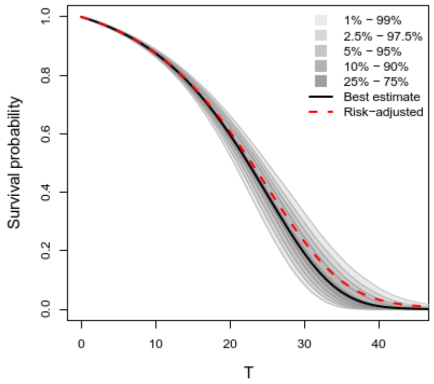
	$x_0 = 50$	$x_0 = 65$	$x_0 = 80$
$\mathbb{E}^{\mathbb{P}}(CoC_H)$	8,330	4,140	1,310
$\mathrm{SD}^{\mathbb{P}}(CoC_H)$	1,920	750	200
corresponding λ	0.315	0.300	0.225

- Risk margin is higher for lower starting ages (longer maturities)
- Considerable uncertainty regarding future capital charges

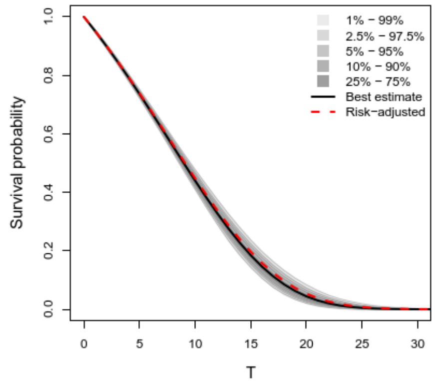
Risk-adjusted forward rates



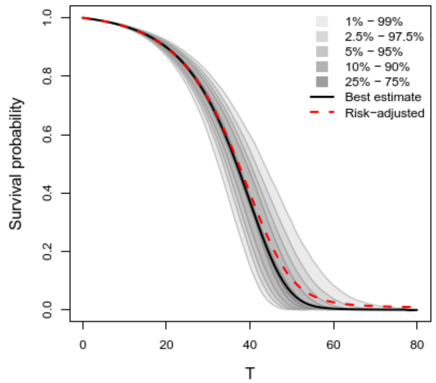
(a) $x_0 = 50$, customized



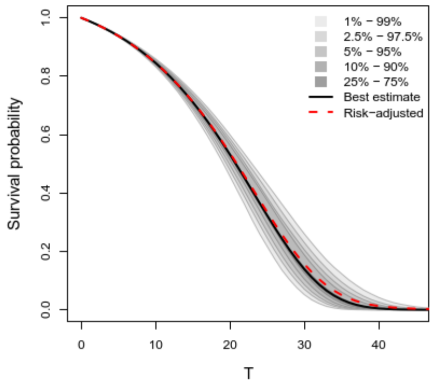
(b) $x_0 = 65$, customized



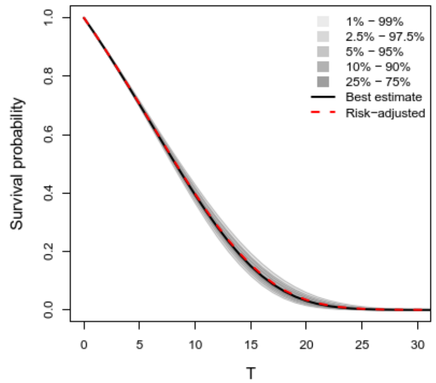
(c) $x_0 = 80$, customized



(d) $x_0 = 50$, index-based



(e) $x_0 = 65$, index-based



(f) $x_0 = 80$, index-based

Risk loadings are

- Higher for customized designs than for index-based designs
- Higher for lower starting ages (longer maturities)

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Conclusion

- **Summary**
  - Framework for pricing longevity-linked securities which accounts for the risk of unanticipated changes in the long-term mortality trend
  - Clear distinction between different components of longevity risk
  - Applicable to both customized and index-based instruments
  - Calibration of the market price of longevity risk
- **Key findings**
  - In the presence of mortality trend changes most of the risk premium is allocated to longer maturities
  - Future Capital charges for longevity risk are subject to a considerable degree of uncertainty
    - Interesting implications for longevity risk management

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