Analysis of tontines from the insurer's perspective

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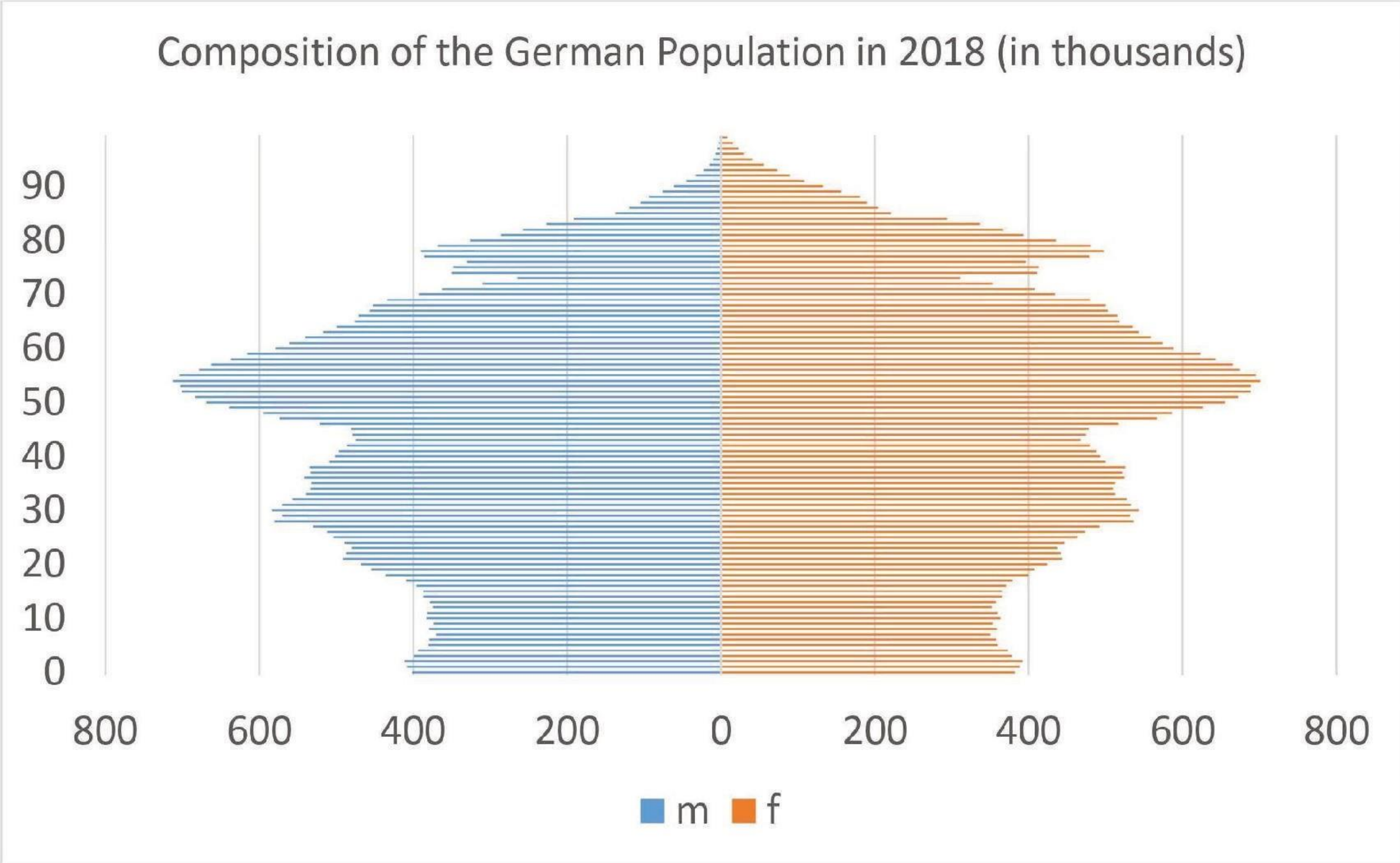
Motivation



Data taken from Statistisches Bundesamt (Destatis) (2019).

Motivation

- Low interest rates, changing demographics and tightening solvency regulation lead to an **increased awareness of the risks contained in retirement products**.
- Innovative products: Group self-annuitization, pooled annuity funds and tontines (Piggott et al. (2005), Valdez et al. (2006), Stamos (2008), Sabin (2010), Donnelly et al. (2013, 2014) and Milevsky and Salisbury (2015)).

(Dis)advantages of annuities and tontines

	Annuity	Tontine
Policyholder	• Stable payments • High prices	• Volatile payments • Cheaper than annuity
Insurer	• High risk capital requirement • Low demand ("Annuity Puzzle")	• Lower risk capital requirement • Higher demand?

Objectives

- Today's focus: **Insurer's perspective**
- To make tontines appealing for insurers, **fees** may be charged to administrate tontines.
- Goals of the paper:
  - Compare **different fee structures**
  - Determine the **critical fee** which makes policyholders indifferent between an annuity and a tontine
  - Analyze **quantities of interest** to the insurer under this critical fee

Selected results

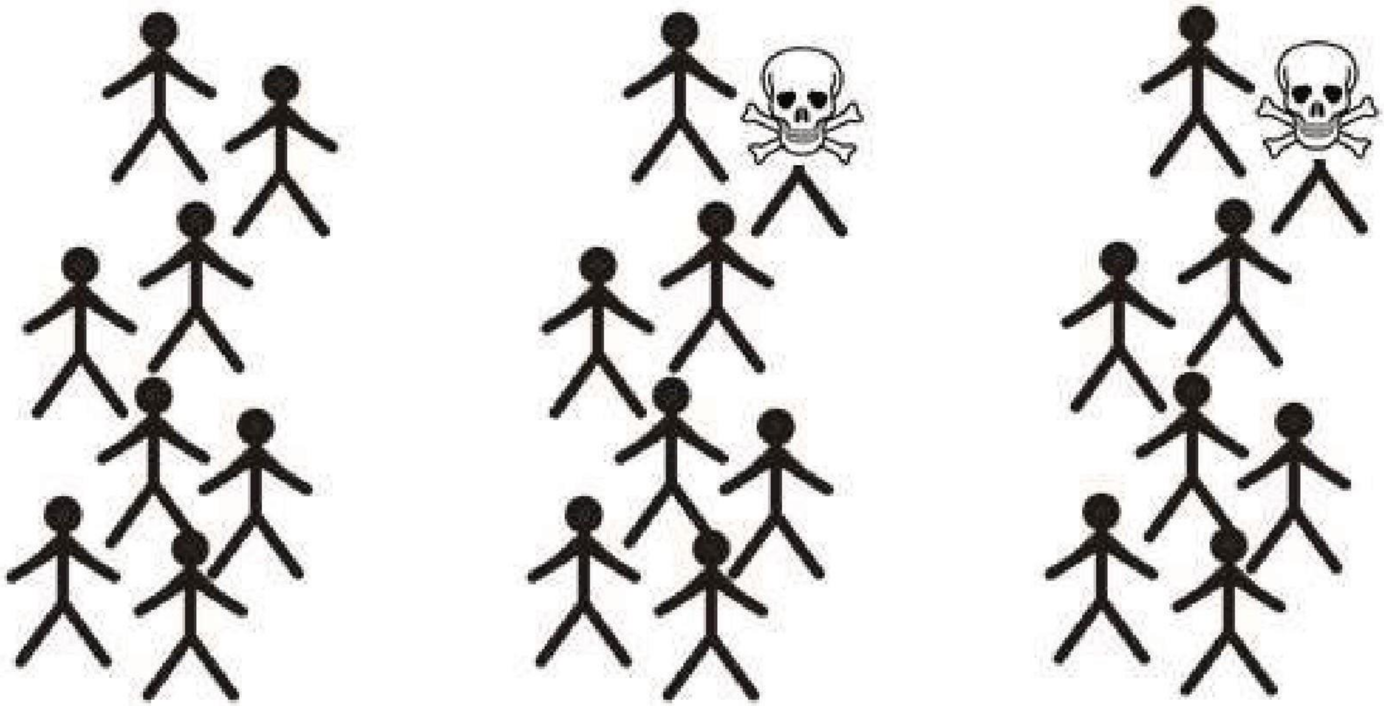
- Policyholders are **indifferent between a single up-front fee and a fixed percentage** being deducted from the retirement benefits over time if the initial values of both fees are identical.
- Sabin (2010) writes that annuities are 14% higher than fair. Given such an annuity, the insurer may charge a fee of up to 12.5% for a tontine from the policyholder.
- Tontines are a lot less volatile than annuities, i.e. the fee is an **almost certain profit**.

Annuity and Tontine

- Following Yaari (1965), we consider continuous-time payment streams.
- ζ is the residual lifetime of the considered individual.
- Annuity: $b_A(t) = \mathbb{1}_{\{\zeta>t\}} c(t)$
  - $c(t)$ is a deterministic function.
- Tontine: $b_{OT}(t) = \mathbb{1}_{\{\zeta>t\}} \frac{n}{N(t)} d(t)$
  - $d(t)$ is a deterministic function.
  - n is number of initial homogeneous policyholders.
  - $N(t)$ is the number still alive at time t.

Example in discrete time

1st year	**2nd year**	**3rd year**
$d(1) = 800, N(1) = 8$	$d(2) = 800, N(2) = 7$	$d(3) = 720, N(3) = 7$
$nd(1)/N(1) = 800$	$nd(2)/N(2) \approx 914$	$nd(3)/N(3) \approx 823$



Mortality risk

Unsystematic mortality risk

- Stems from the fact that the lifetime of a person is unknown but still follows some certain mortality law.
- Can initially be diversified by a large pool size

Systematic mortality risk

- Stems from the fact that the true mortality law cannot be determined explicitly.
- Cannot be diversified as it affects the pool as a whole

Mortality risk

- ${}_tp_x$ is the t-year survival probability of an x-year old.
- Apply random longevity shock ϵ with values in $(-\infty, 1)$ to obtain ${}_tp_x^{1-\epsilon}$
- f_ϵ and m_ϵ are the density and the moment-generating function of ϵ.
- ζ_ϵ and $N_\epsilon(t)$ depend on the shock ϵ.

Two fee structures

- **Fixed initial** fee M_0/n subtracted at the beginning:

$$b^F_{OT}(t) := \mathbb{1}_{\{\zeta_\epsilon > t\}} \frac{nd^F(t)}{N_\epsilon(t)}.$$

- **Time-varying proportional** fee $\alpha(t)$ subtracted over time:

$$b^V_{OT}(t) := \mathbb{1}_{\{\zeta_\epsilon > t\}} \frac{(1-\alpha(t))nd^V(t)}{N_\epsilon(t)}.$$

Premium calculation

- Let r be the constant risk-free interest rate.
- Premium under **fixed initial fee** M_0/n:

$$P_0^F = \mathbb{E}\left[\int_0^\infty e^{-rt} b_{OT}^F(t)\mathrm{d}t\right]$$

$$= \int_0^\infty e^{-rt}\, d^F(t) \int_{-\infty}^{1} \left(1 - \left(1 - {}_tp_x^{1-\varphi}\right)^n\right) f_\epsilon(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}t$$

$$\widetilde{P}_0^F = P_0^F + \frac{M_0}{n}$$

Premium calculation

Premium under **time-varying proportional fee** $\alpha(t)$:

$$P_0^V = \mathbb{E}\left[\int_0^\infty e^{-rt} b_{OT}^V(t)\mathrm{d}t\right]$$

$$= \int_0^\infty e^{-rt}(1-\alpha(t))d^V(t)\int\limits_{-\infty}^{1}\left(1-\left(1-{}_tp_x^{1-\varphi}\right)^n\right)f_\epsilon(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}t$$

$$\widetilde{P}_0^V = P_0^V + \int_0^\infty e^{-rt}\alpha(t)d^V(t)\int\limits_{-\infty}^{1}\left(1-\left(1-{}_tp_x^{1-\varphi}\right)^n\right)f_\epsilon(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}t$$

Optimization problem under fixed initial fee

- Consider a retiree endowed with an initial wealth $v > 0$, a utility function $u(y) = \frac{y^{1-\gamma}}{1-\gamma}$, $\gamma > 1$, $\gamma \neq 1$ and a subjective discount factor ρ.

- At time 0, the **policyholder solves**:

$$\max_{d^F(t)} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u\left(\frac{nd^F(t)}{N_\epsilon(t)}\right) \mathbb{1}_{\{\zeta_\epsilon > t\}} \mathrm{d}t\right]$$

$$\text{subject to } P_0^F + \frac{M_0}{n} \leq v$$

- To solve this (explicitly), rearrange the budget constraint to $P_0^F \leq v - \frac{M_0}{n}$ and apply Theorem 2 in Chen et al. (2019).

Optimization problem under time-varying proportional fee

- At time 0, the **policyholder solves**:

$$\max_{d^V(t)} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u\left(\frac{n(1-\alpha(t))d^V(t)}{N_\epsilon(t)}\right) \mathbb{1}_{\{\zeta_\epsilon > t\}} \mathrm{d}t\right] \text{ subject to}$$

$$P_0^V + \int_0^\infty e^{-rt} \alpha(t) d^V(t) \int_{-\infty}^{1} \left(1 - \left(1 - {}_tp_x^{1-\varphi}\right)^n\right) f_\epsilon(\varphi)\, \mathrm{d}\varphi\, \mathrm{d}t \leq v$$

- Explicit solution:

$$d^{V*}(t) = \frac{e^{\frac{(r-\rho)t}{\gamma}} (1-\alpha(t))^{1/\gamma - 1} \left(\kappa_{n,\gamma,\epsilon}({}_tp_x)\right)^{1/\gamma}}{\lambda_V^{1/\gamma} \left(\int_{-\infty}^{1} \left(1 - \left(1 - {}_tp_x^{1-\varphi}\right)^n\right) f_\epsilon(\varphi)\, \mathrm{d}\varphi\right)^{1/\gamma}}.$$

Optimization problem under time-varying proportional fee

The optimal Lagrangian multiplier λ_V is given by

$$\lambda_V = \left(\frac{1}{v} \left(\int_0^\infty e^{\left(\frac{1}{\gamma}-1\right) rt - \frac{1}{\gamma}\rho t} \cdot \frac{(1-\alpha(t))^{1/\gamma-1} \left(\kappa_{n,\gamma,\epsilon}({}_t p_x)\right)^{1/\gamma}}{\left(\int_{-\infty}^1 \left(1-\left(1-{}_t p_x^{1-\varphi}\right)^n\right) f_\epsilon(\varphi)\,\mathrm{d}\varphi \right)^{1/\gamma-1}} \,\mathrm{d}t \right) \right)^\gamma,$$

where

$$\kappa_{n,\gamma,\epsilon}({}_t p_x) = \sum_{k=1}^n \binom{n}{k} \left(\frac{k}{n}\right)^\gamma \int_{-\infty}^1 \left({}_t p_x^{1-\varphi}\right)^k \left(1 - {}_t p_x^{1-\varphi}\right)^{n-k} f_\epsilon(\varphi)\,\mathrm{d}\varphi.$$

The **optimal level of expected utility** is given by

$$U_V = \frac{1}{1-\gamma} \cdot \lambda_V \cdot v.$$

Comparison of the fee structures

- Is there a **preferable fee structure** for the policyholder?
- To compare the fee structures, it shall hold:

$$M_0 = \int_0^\infty e^{-rt}\,\alpha(t)\,n\,d^V(t) \int\limits_{-\infty}^{1} \left(1 - \left(1 - {}_tp_x^{1-\varphi}\right)^n\right) f_\epsilon(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}t.$$

- Under this assumption and if $\alpha(t) = \alpha$, it holds $U_V = U_F$.
- A decreasing fee $\alpha(t)$ results (numerically) in $U_V < U_F$.

Gompertz law (Gompertz (1825))

- For a modal age at death $m > 0$ and a dispersion coefficient $\beta > 0$, the force of mortality is, for any x and $t \geq 0$, given by

$$\mu_{x+t} = \frac{1}{\beta} e^{\frac{x+t-m}{\beta}}.$$

- This implies that the t-year survival probability of an x-year old is given by

$$_{t}p_{x} = e^{e^{\frac{x-m}{\beta}}\left(1-e^{\frac{t}{\beta}}\right)}.$$

Parameter setup

Initial wealth $v = 100$	Pool size $n = 1000$	Risk aversion $\gamma = 4$
Fee $M_0 = 5000$	Risk-free rate $r = 0.01$	Subjective discount rate $\rho = r$
Initial age $x = 65$	Gompertz law $m = 88.721, \beta = 10$	Longevity shock $\epsilon \sim \mathcal{N}_{(-\infty,1)}\left(-0.0035, 0.0814^2\right)$

Table: Base case parameter setup. A pool size of $n = 1000$ is used e.g. in Qiao and Sherris (2013), m and β are chosen as in Milevsky and Salisbury (2015), the parameters of the shock are taken from Chen et al. (2019) and the risk-free interest rate is suggested by Statista (2019).

Numerical example

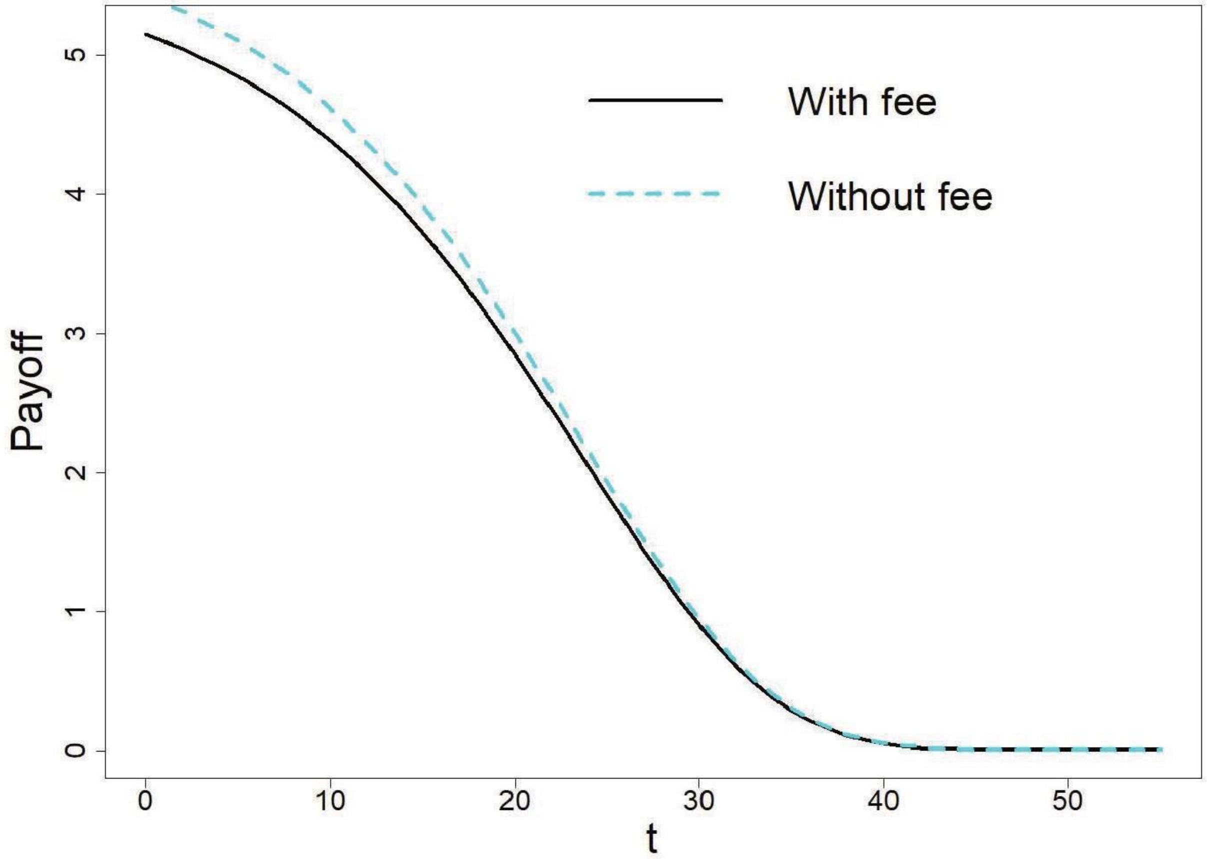


Figure: Optimal payoff for two fee levels $M_0/n = 5$ and $M_0/n = 0$.

Indifference Fee

- For a given annuity fee Δ_0, how high is the **maximum tontine fee** the insurer may charge?
- The indifference fee of the tontine is chosen such that the policyholder is **indifferent** between an annuity and a tontine.
- Indifference fee M_0^*/n is defined by

$$\lambda_F\left(v - \frac{M_0^*}{n}\right)$$
$$= (v - \Delta_0)^{1-\gamma}\left(\int_0^\infty e^{\left(\frac{1}{\gamma}-1\right)rt - \frac{1}{\gamma}\rho t}\int_{-\infty}^1 {}_tp_x^{1-\varphi} f_\epsilon(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}t\right)^\gamma.$$

Indifference fee

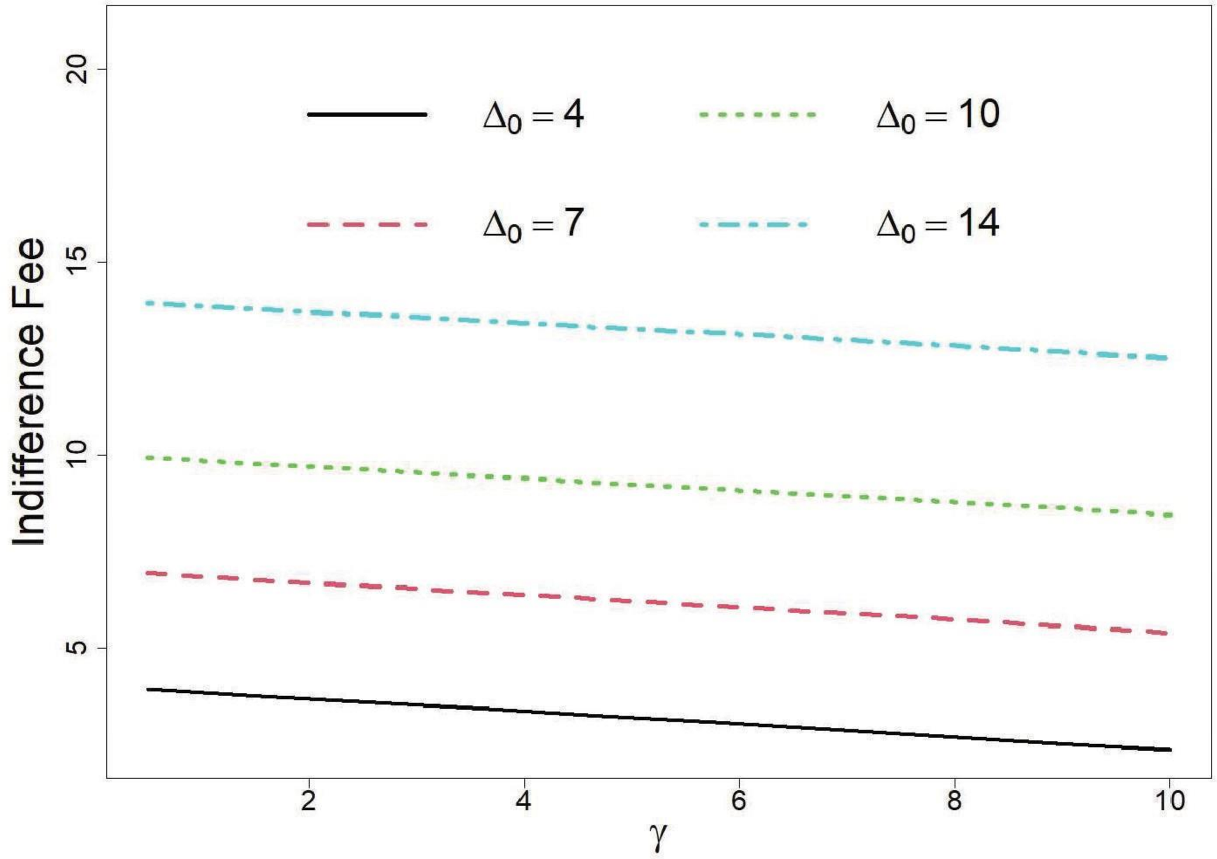


Figure: Indifference fee of the tontine in dependence of the relative risk aversion γ. The fee levels of the annuity are based on Chen et al. (2019) and Sabin (2010).

Mean and variance analysis

- Expected profit at time 0 of the annuity is **higher** than that of the tontine:

$$\Delta_0 > M_0^*/n.$$

- Tontine payoff from the insurer's perspective:

$$B_{OT}(t) = nd(t)\mathbb{1}_{\{N_\epsilon(t)>0\}}.$$

- Annuity payoff from the insurer's perspective:

$$B_A(t) = c(t)N_\epsilon(t).$$

Variance analysis

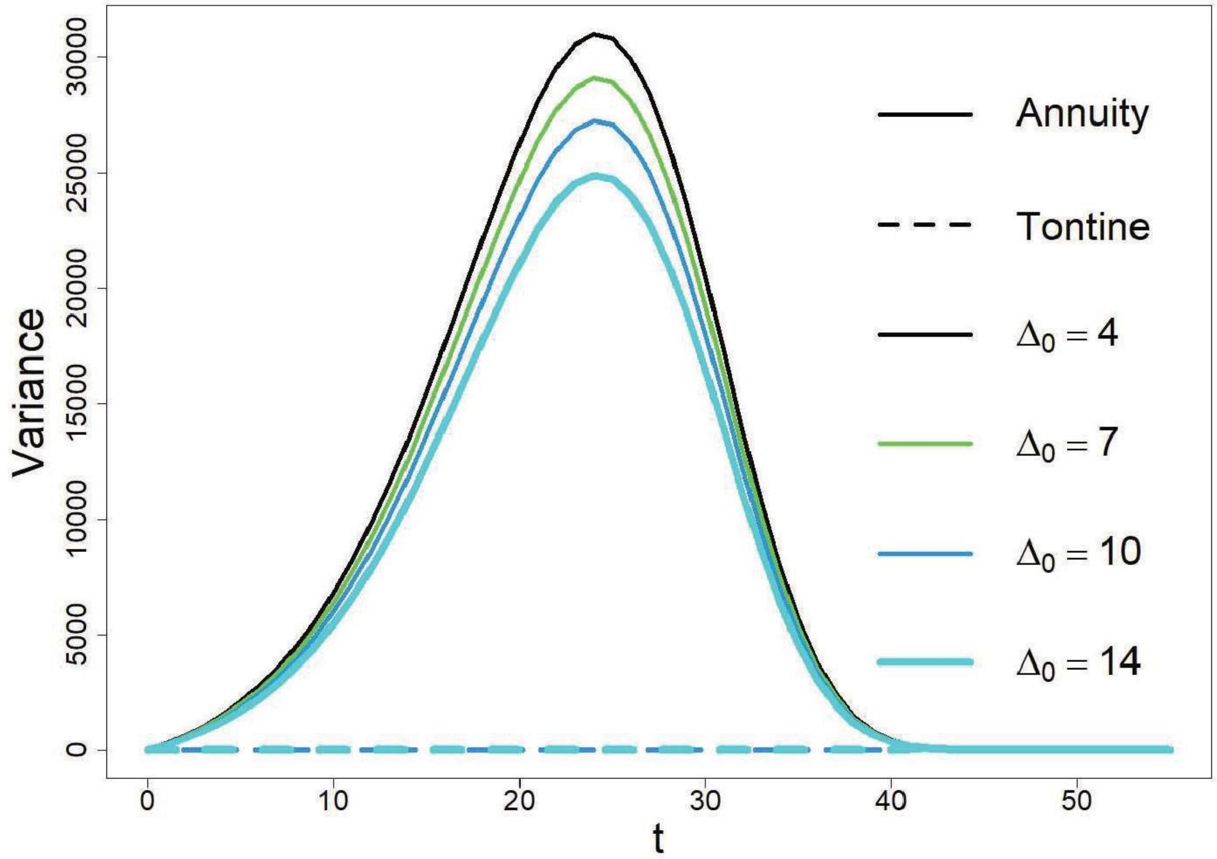


Figure: Variance of the annuity and tontine payoffs $\mathrm{Var}(B_A(t))$ and $\mathrm{Var}(B_{OT}(t))$ from the insurer's perspective over time. The fee charged for the tontine is the indifference fee.

Coefficient of variation analysis

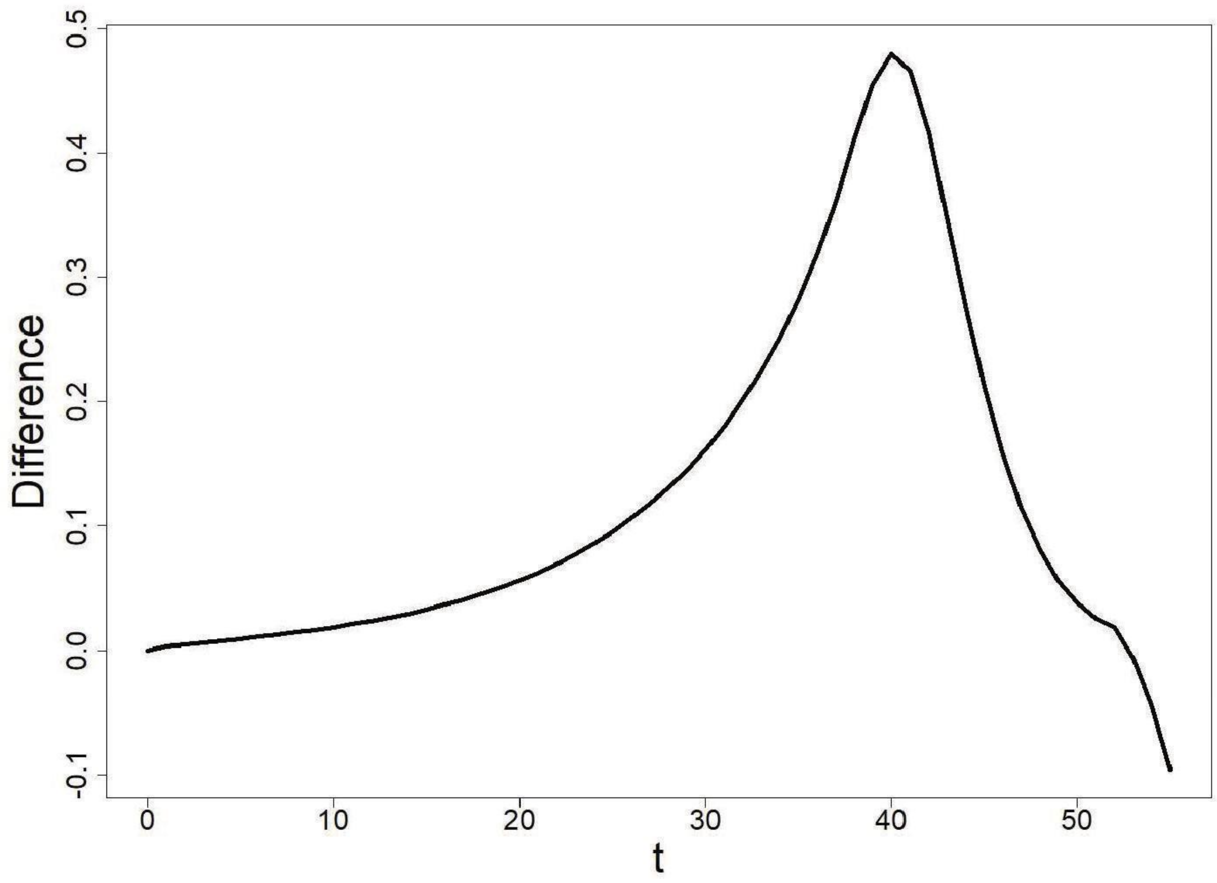
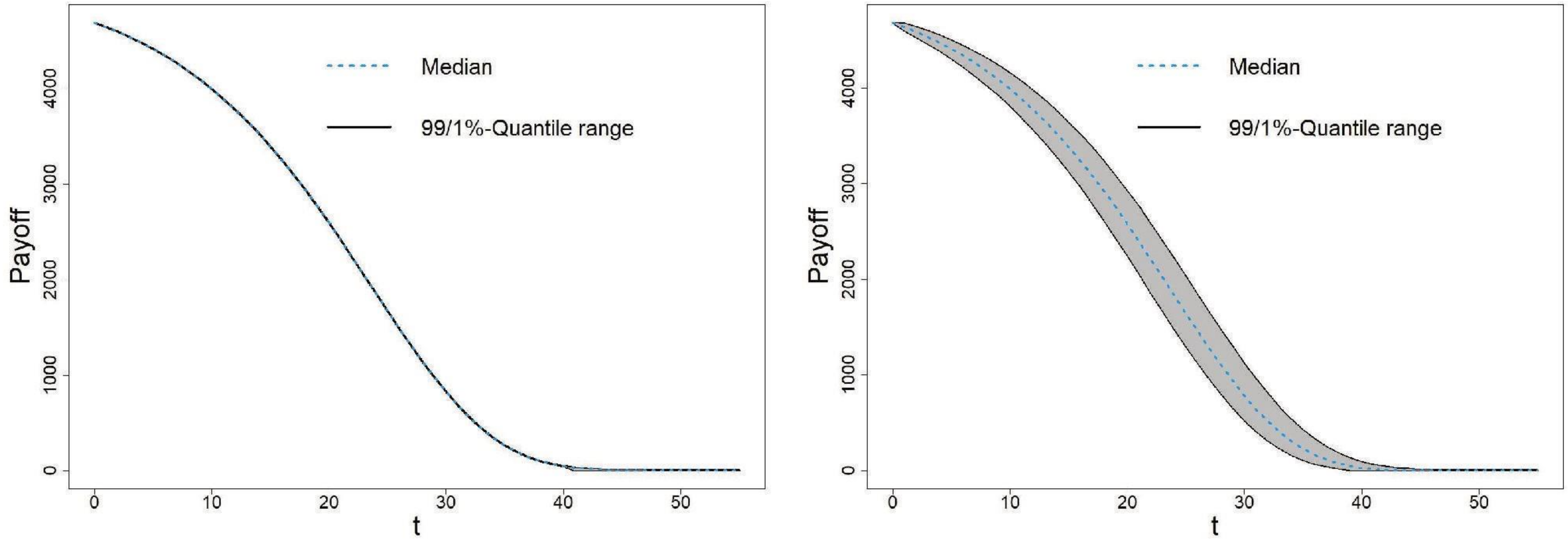


Figure: Difference of the coefficients of variation of the annuity and tontine payoffs $\mathrm{CV}(B_A(t)) - \mathrm{CV}(B_{OT}(t))$ from the insurer's perspective over time.

Quantile analysis



(a) Tontine (b) Annuity

Figure: Quantiles of the annuity and tontine payoffs from the insurer's perspective over time. The fee charged for the annuity is $\Delta_0 = 14$. The fee charged for the tontine is the indifference fee which is equal to 13.42 for $\gamma = 4$.

Analysis of the reserves

- Following Börger (2010) and Chen et al. (2019), we assume that mortality evolves according to best-estimate assumptions.
- Reserve of the tontine:

$$
\begin{aligned}
{}_tV_x^{OT} = n_t p_x \int_t^{\infty} & e^{-r(s-t)} \\
& \cdot \int_{-\infty}^{1} \left(1 - (1 - {}_{s-t}p_{x+t})^n\right) f_\epsilon(\varphi)\mathrm{d}\varphi \cdot d(s)\,\mathrm{d}s
\end{aligned}
$$

- Reserve of the annuity:

$$
{}_tV_x^{A} = n_t p_x \int_t^{\infty} e^{-r(s-t)}{}_{s-t}p_{x+t} \cdot m_\epsilon(-\ln {}_{s-t}p_{x+t})c(s)\,\mathrm{d}s.
$$

Analysis of the reserves

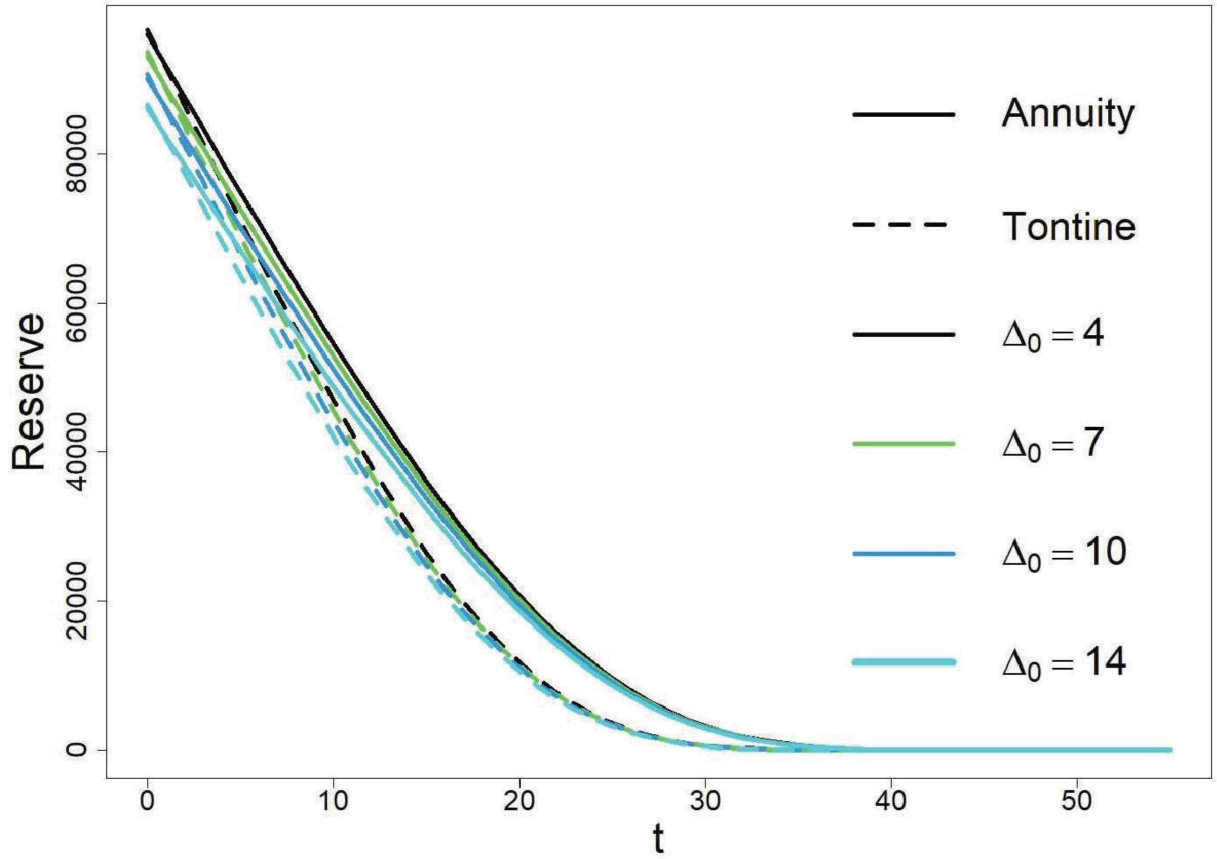


Figure: Reserves of the annuity and tontine over time. The fee charged for the tontine is the indifference fee.

Summary

- Policyholders are **indifferent between a single up-front fee and a fixed percentage** being deducted from the retirement benefits over time if the initial values of both fees are identical.
- Insurers may charge a fee close to that of annuities for tontines from the policyholder.
- Tontines are a lot less volatile than annuities, i.e. the fee is an **almost certain profit**.

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