



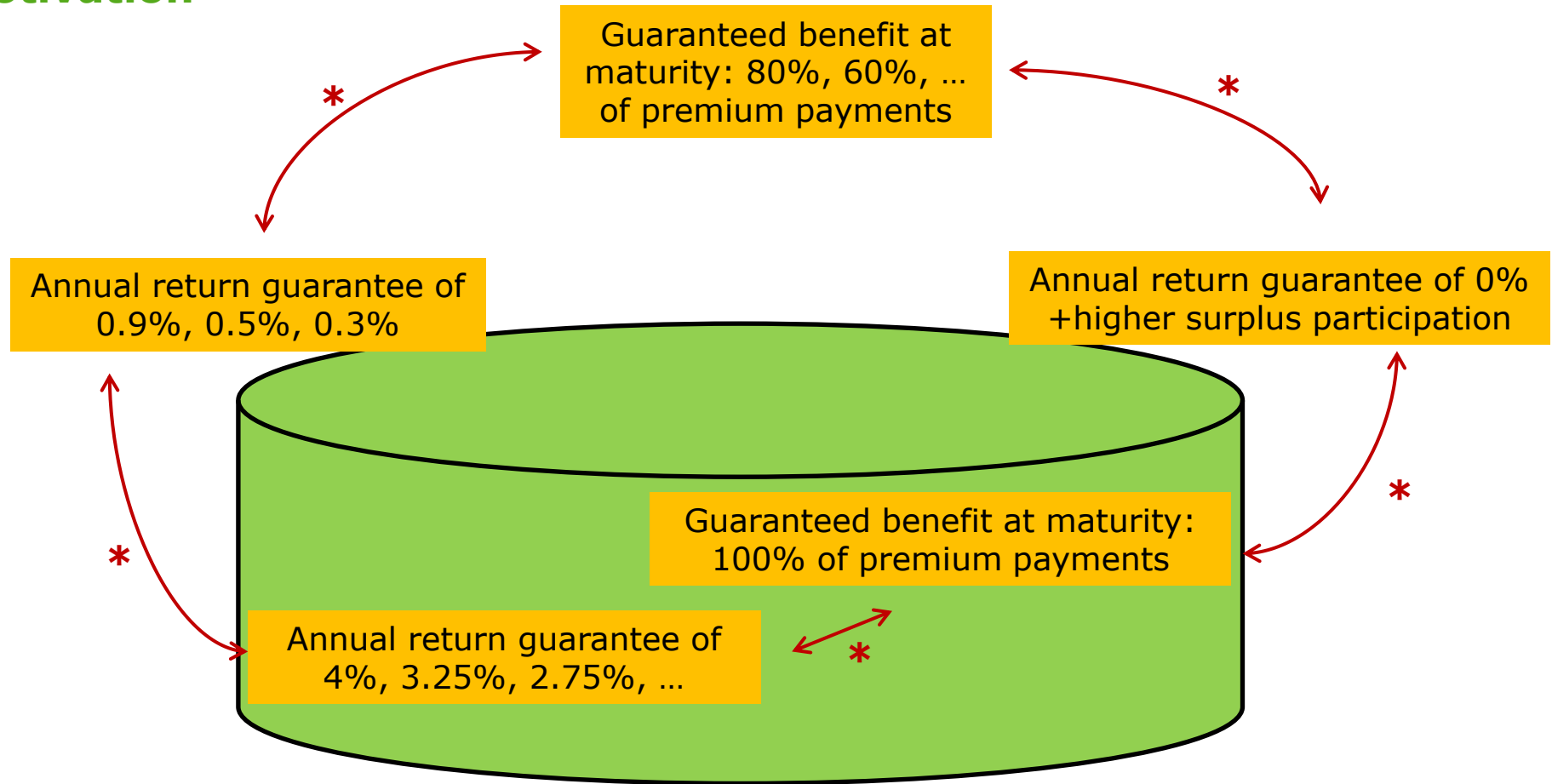
# **The interaction of contracts in a heterogeneous life insurance portfolio**

Effects of a differentiation of the surplus participation

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## Motivation



## Life insurance portfolio

\* investing the premiums in the same asset portfolio; sharing investment returns of a joint asset pool and sharing the same bonus reserves

## Agenda

### **How can you measure the interaction of contracts in a heterogeneous life insurance portfolio?**

The concept of the collective bonus

### **What are the effects of a differentiation of surplus participation on existing policyholders, new policyholders and the shareholders?**

Analysis framework

Pricing strategies of the new business

Effects of the new business strategies

Conclusion

## The concept of the collective bonus

### Definition of the collective bonus

- The concept of the collective bonus measures how much a contract **will in expectation (ex ante) benefit** or **has actually benefited (ex post)** from being part of the insurance collective.
- The collective bonus of contract  $i$  at time  $t^*$  (valuation date) consists of two parts:

$$CB_{t^*}^i = CB_{t^*}^i(\text{ex ante}) + CB_{t^*}^i(\text{ex post})$$

#### Ex ante collective bonus

$$CB_{t^*}^i(\text{ex ante}) = E_Q \left[ \sum_{j=t^i}^{t_0^i+T^i-1} B_{j+1}^{-1} \cdot L_j^i \cdot \left( 1 + f_{j+1}^i - \frac{F_{j+1}}{F_j} \right) \right]$$

#### Ex post collective bonus

$$CB_{t^*}^i(\text{ex post}) = L_{t^*}^i - \sum_{j=t_0^i}^{t^*} P_j^i \cdot \prod_{k=j+1}^{t^*} \frac{F_k}{F_{k-1}}$$

- Notations:

- $L_{t^*}^i$ : policyholder account of contract  $i$  at the valuation date  $t^*$ ; conclusion  $t_0^i$ ; term of  $T^i$
- $P_t^i$ : premium payment of contract  $i$  at time  $t$ ; invested in the same asset portfolio  $(F_t)_{t \geq t_0^i}$ .
- $\frac{F_t}{F_{t-1}}$ : net market return of the asset portfolio in year  $t$ ; risk-free asset  $(B_t)_{t \geq 0}$
- $f_t^i$ : contractually arranged credit of contract  $i$  at time  $t$

- The collective bonus is the difference of the **contract's return** and **a theoretical investment of the contract's premiums in the company's asset portfolio at market value.**

## The concept of the collective bonus

### Definition of the collective bonus

– Collective bonus of the shareholders:

- **Ex ante:** present value of future profits represents all future gains or losses of the shareholders ( $T$  is the expiration of the last contract of the insurance company):

$$CB_{t^*}^{sh}(\text{ex ante}) = PVFP_{t^*} = E_Q \left[ \sum_{j=t^*+1}^T B_j^{-1} \cdot X_j \right]$$

- **Ex post:** all previous cash flows  $X_j$ ,  $j \leq t^*$ , had been invested in the company's asset portfolio

$$CB_{t^*}^{sh}(\text{ex post}) = \sum_{j=t_0^1+1}^{t^*} X_j \cdot \prod_{l=j+1}^{t^*} \frac{F_l}{F_{l-1}}$$

- The so-called **best estimate of liabilities**  $BE_{t^*}^i$  of cohort  $i$  at time  $t^*$  is defined as the present value of future expected cash flows payable to the policyholders less the present value of future premium payments under the equivalent pricing measure  $Q$

$$BE_{t^*}^i = E_Q \left[ B_{t_0^i+T^i}^{-1} \cdot L_{t_0^i+T^i}^i - \sum_{j=t^*+1}^{t_0^i+T^i-1} B_j^{-1} \cdot P_j^i \right]$$

- The best estimate of liabilities at time  $t^*$  of all cohorts is denoted by  $BE_{t^*}$ .

## The concept of the collective bonus

### Definition of the collective bonus

- The sum of the collective bonuses of the shareholders ( $CB_{t^*}^{sh}$ ) and the policyholders ( $CB_{t^*}$ ) is zero if and only if the market value of assets ( $A_{t^*}$ ) is equal to  $BE_{t^*} + PVFP_{t^*}$ , i.e.

$$A_{t^*} = BE_{t^*} + PVFP_{t^*} \Leftrightarrow CB_{t^*} + CB_{t^*}^{sh} = 0$$

- If the market value of assets is equal to  $BE_{t^*} + PVFP_{t^*}$  and one cohort of contracts has a non-zero collective bonus, the other cohorts or the shareholders have to compensate for this difference.
- If contract  $i$  is concluded at the valuation date  $t^*$ , i.e.  $t_0^i = t^*$ , we get

$$CB_{t^*}^i(\text{ex ante}) = BE_{t^*}^i = E_Q \left[ B_{t_0^i+T^i}^{-1} \cdot L_{t_0^i+T^i}^i - \sum_{j=t^*+1}^{t_0^i+T^i-1} B_j^{-1} \cdot P_j^i \right]$$



The concept of the collective bonus provides a well-defined method to systematically analyse the interaction of contracts in a heterogeneous life insurance portfolio, in particular with respect to cross-subsidizing effects.

## Agenda

### **How can you measure the interaction of contracts in a heterogeneous life insurance portfolio?**

The concept of the collective bonus

### **What are the effects of a differentiation of surplus participation on existing policyholders, new policyholders and the shareholders?**

Analysis framework

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## Motivation

- **2004:** The “Bundesanstalt für Finanzdienstleistungsaufsicht (BaFin)” prohibited\* the differentiation of surplus participation in the case of a pooled capital investment.
  - §138 Absatz 2 VAG: *„Bei gleichen Voraussetzungen dürfen Prämien und Leistungen nur nach gleichen Grundsätzen bemessen werden.“*
- **2017:** “Deutsche Aktuarvereinigung e.V. (DAV)” argues\*\* that the pronouncement of the “BaFin” cannot be applied in the current situation due to significantly different conditions
  - No opportunity to choose between products with different kind of guarantees in 2004
  - Maximum actuarial interest rate was approximately equal for all cohorts of insurance contracts in Germany in 2004and therefore a differentiation of surplus participation can be justified.
- Many insurance companies follow the report of the DAV and currently allow for a differentiation of the surplus participation to compensate for example lower guarantees:
  - DAV (2017) provides an instruction to determine a “spread”, which indicates the differentiation of surplus participation.

\* Hinweise zur Wahrung des Gleichbehandlungsgrundsatzes bei der Verteilung der Überschüsse an die Versicherungsnehmer (Bonn, Juli 2004)

\*\* Ergebnisbericht des Ausschusses Lebensversicherung: Aktuarielle Anmerkungen zur Differenzierung der Überschussbeteiligung (Köln, November 2017)



## Analysis framework

### Cash flow model, financial market model and asset model

#### – Cash flow model

- Twenty different cohorts of traditional participating endowment policies\*:
  - Guaranteed maturity benefit plus ongoing bonus and terminal bonus
  - Cohorts 1 to 19 represent existing portfolio:  
 $g^i$  = maximum actuarial interest rate in Germany of the corresponding year
  - Cohort 20 represents the new business

#### – Financial market model

- parameter calibration follows so called "Branchensimulationsmodell"\*\*\*
- Short rate process: Hull White model; stock price: geometric Brownian motion

#### – Asset model

- 10% stocks and 90% bonds with fixed initial maturity
- Rebalancing at the end of the year in terms of market values

\*Based on model in Burkhart et al. (2017): Allowance for surplus funds under Solvency II: adequate reflection of risk sharing between policyholders and shareholders in a risk-based solvency framework?

\*\*\*German standard valuation model for Solvency II developed by the German Association of Insurance Companies (GDV).

## Analysis framework

### Balance sheet and liability model

- Simplified **balance sheet** at time  $t$  (German accounting principles; HGB)
  - *Asset side*: book value of the stocks and bonds
  - *Liability side*: shareholders' equity, the free reserve for bonuses and rebates (*fRfB*), terminal bonus funds (*TBF*), actuarial reserve and bonus reserve (*BR*).
- **Surplus participation**: Declared bonus is split into an ongoing ( $^{acc}bon_{t+1}$ ) and a terminal bonus ( $^{term}bon_{t+1}$ ):
  - *The investment boni are distributed such that all **existing policyholders** receive the same total yield on their accounts value. The **new business cohort** possibly gets a higher bonus rate (=bonus rate (existing business) + spread).*
  - *All policyholders receive at least the guaranteed interest rate. If investment boni are not sufficient for all policyholders to receive at least the guaranteed interest rate, the bonus rates of cohorts with a lower guaranteed interest rate are reduced accordingly.*
- Ongoing bonus (guaranteed interest rate also applies to the bonus reserve):

$$BR_t^i = BR_{t-1}^i \cdot (1 + g^i) + ^{acc}bon_t^i$$

- Terminal bonus:

$$TBF_t^i = TBF_{t-1}^i + ^{term}bon_t^i$$

## Pricing strategies of the new business

- **Strategy A** (no reduction of the guaranteed interest; no spread):
  - Guaranteed interest is 0.9% p.a.
  - No differentiation of the surplus participation
- **Strategy B** (reduction of the guaranteed interest; no spread):
  - Guaranteed interest is 0.0% p.a.
  - No differentiation of the surplus participation
- **Strategy C** (reduction of the guaranteed interest; individual spread):
  - Guaranteed interest is 0.0% p.a.
  - Individual spread determination
- **Strategy D** (reduction of the guaranteed interest; collective spread):
  - Guaranteed interest is 0.0% p.a.
  - Collective spread determination
- **Strategy E**: no new business

## Strategy C: Determination of the individual spread

- **Individual** determination: only new business and shareholders interact
- First valuation (strategy A:  $g^{NB} = 0.90\%$ ;  $s^A = 0\%$ ):

$$E_Q \left[ L_{20}^{NB}(g^{NB} = 0.90\%; s^A = 0\%) \cdot B_{20}^{-1} - \sum_{j=0}^{19} P_j^{NB}(g^{NB} = 0.90\%) \cdot B_j^{-1} \right]$$

**= \***

- Second independent valuation:

$$E_Q \left[ L_{20}^{NB}(g^{NB} = 0.00\%; s^C) \cdot B_{20}^{-1} - \sum_{j=0}^{19} P_j^{NB}(g^{NB} = 0.00\%) \cdot B_j^{-1} \right]$$

- **Result:** With a spread  $s^C = \mathbf{0.36\%}$ :  $CB_0^{NB}(\text{ex ante; strategy A}) = CB_0^{NB}(\text{ex ante; strategy C}) = 361,798 \text{ €}$

\* Ergebnisbericht des Ausschusses Lebensversicherung: Aktuarielle Anmerkungen zur Differenzierung der Überschussbeteiligung (Köln, 16. November 2017)

## Strategy D: Determination of the collective spread

- **Collective** determination: Interaction with existing portfolio is taken into account.

|            | Guarantee $g^{NB}$ | Spread $s$ | Ex ante CB (individual) | Ex ante CB (collective) |
|------------|--------------------|------------|-------------------------|-------------------------|
| strategy A | 0.90%              | 0.00%      | 361,798 €               | 777,113 €               |
| strategy C | 0.00%              | 0.36%      | 361,798 €               | 336,901 €               |

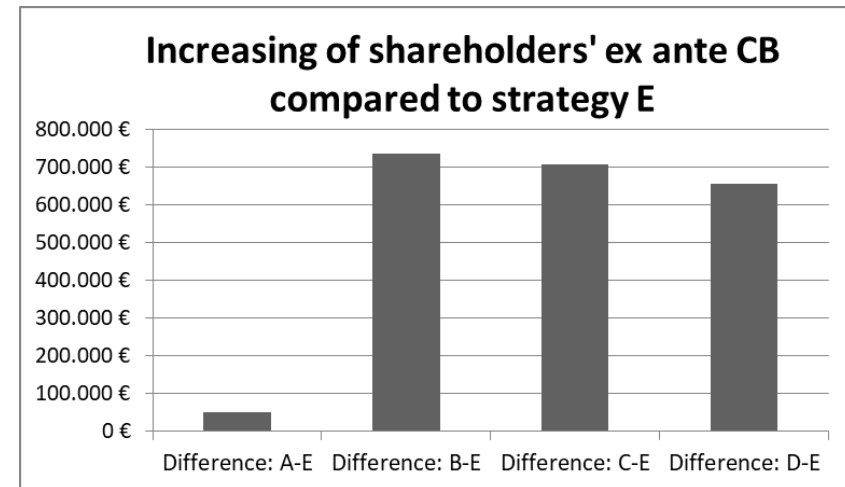
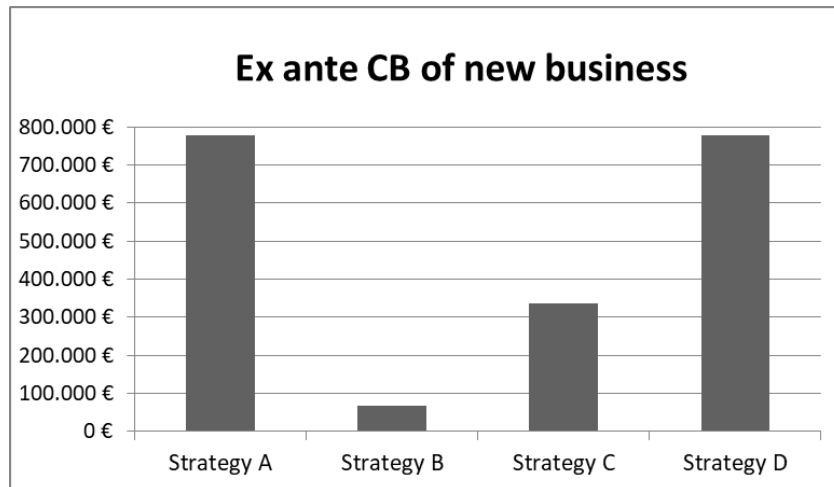
- *Individual determination*: strategy A and strategy C are of “equal value”
- *Collective determination*: ex ante CB of strategy A is greater than ex ante CB of strategy C
- The concept of the collective bonus is again used to calculate a fair spread (this time in a collective way) and we get
  - With a spread  $s^D = \mathbf{0.97\%}$ :  $CB_0^{NB}(\text{ex ante; strategy A}) = CB_0^{NB}(\text{ex ante; strategy D}) = 777,113 \text{ €}$



The concept of the collective bonus provides a systematic method to determine a *fair* spread for surplus participation.

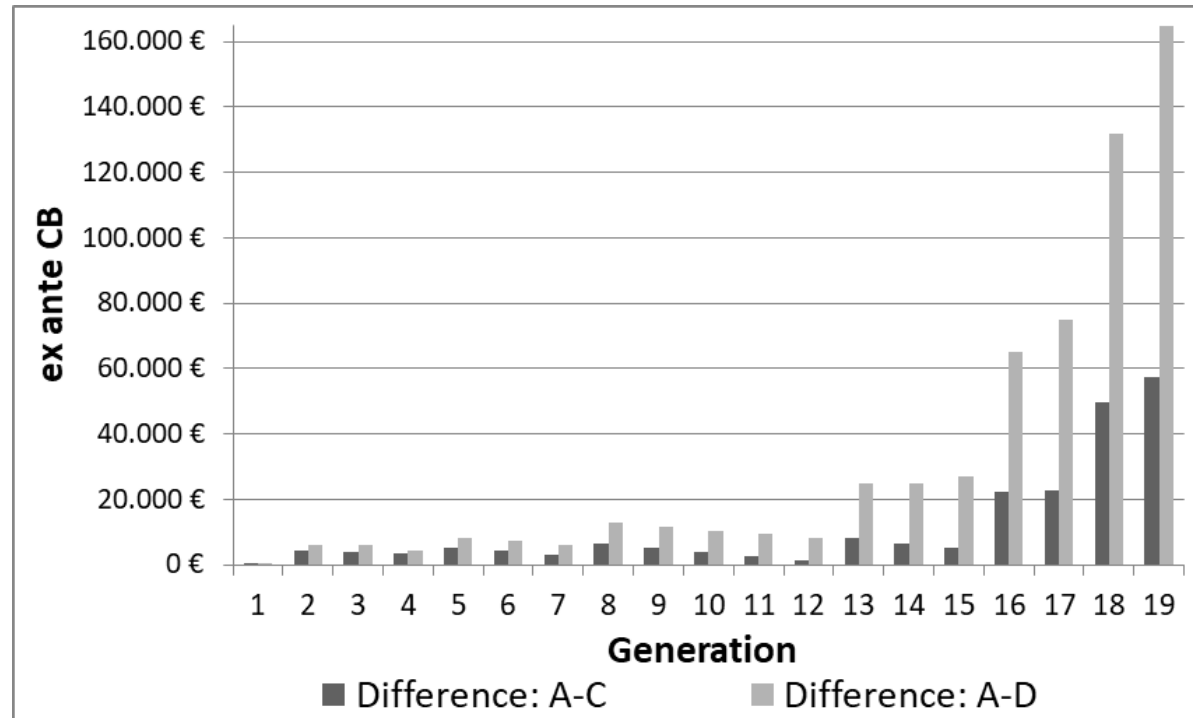
## Effects of the new business strategies on new business and the shareholders

|                    | A             | B             | C             | D             | E               |
|--------------------|---------------|---------------|---------------|---------------|-----------------|
| Guarantee   Spread | 0.90%   0.00% | 0.00%   0.00% | 0.00%   0.36% | 0.00%   0.97% | No new business |
| New business       | 777,113 €     | 67,806 €      | 336,901 €     | 777,113 €     | -               |
| Shareholders       | 6,189,427 €   | 6,873,542 €   | 6,845,306 €   | 6,792,987 €   | 6,138,695 €     |



- *Reduction of guarantee of new business*: new business from 777,113 € (A) to 67,806 € (B) and ex ante CB of shareholders increases by 684,115 € (less guarantee to be paid).
- *Higher surplus participation of new business*: increasing of ex ante CB of existing business and decreasing of ex ante CB of shareholders.

## Effects of the new business strategies on the existing portfolio: Strategy A vs. strategy C and D



- Reduction of ex ante collective bonus of cohort 1 to 19 by increasing the differentiation of surplus participation
  - *Bad performance of the asset portfolio*: guaranteed allocation in A, C and D
  - *Good performance of the asset portfolio*: less surplus participation in C and D than in A
- Cohorts which interact with the new business a long period are most affected.

## First results

- What are the effects of a differentiation of surplus participation on...
  - ... **new policyholders**?
    - A reduction of the guarantee can be compensated by a higher surplus participation.
  - ... **the shareholders**?
    - The expected profit for the shareholders is greater for new business with a lower guaranteed interest rate and higher surplus participation than in the situation with a higher guaranteed interest rate and lower surplus participation.
  - ... **existing policyholders**?
    - The expected attractiveness of the existing business is reduced for a new business strategy with a reduced guarantee and increased surplus participation as compared to a new business strategy without a reduction in the guarantee and without differentiation of the surplus participation.
- How can this decrease in the expected attractiveness of the existing portfolio be compensated?

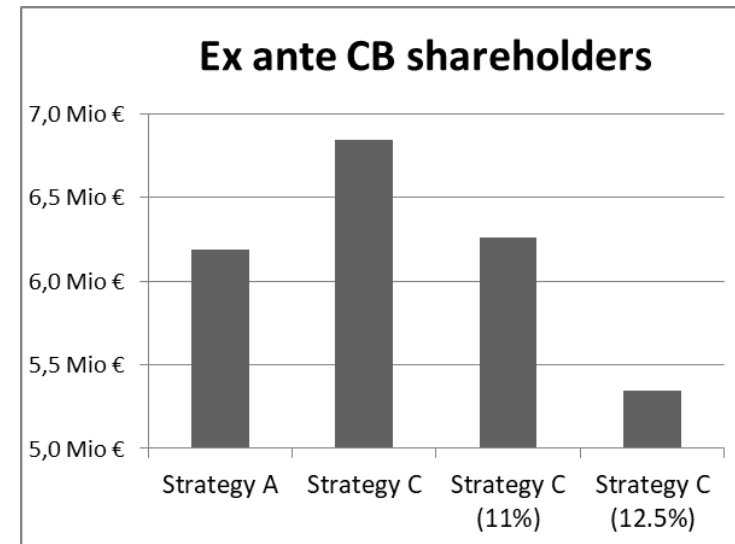
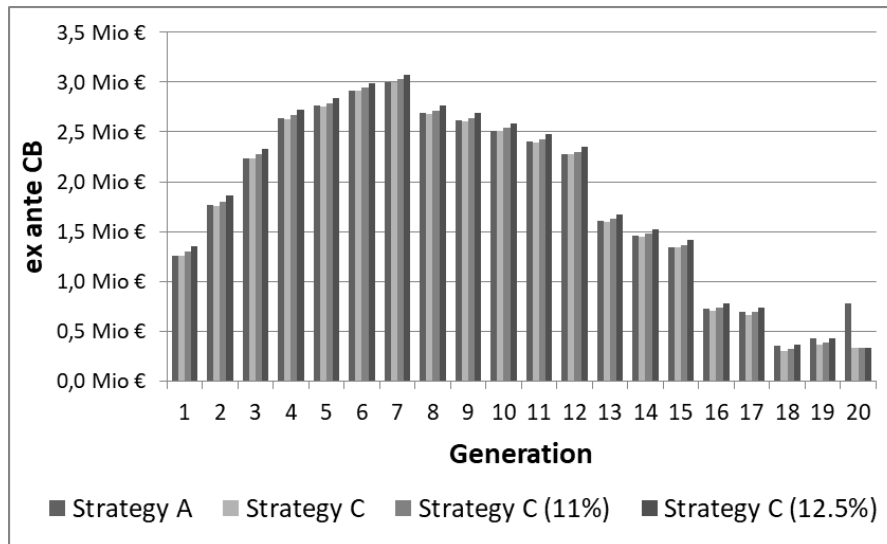


The effects of different new business strategies on the existing portfolio, the new business and shareholders can be systematically analysed using the concept of the collective bonus, especially the effects of a differentiation of surplus participation.



## Effect of the asset allocation on the existing portfolio

- Increasing of the stock ratio (10%) to 11% (strategy C (11%)) and 12.5% (strategy C (12.5%)).
  - Individual spread of strategy C (11%): 0.35%
  - Individual spread of strategy C (12.5%): 0.33%



- The ex ante CB of the existing portfolio and shareholders in strategy C (11%) are greater than in strategy A.



The concept of the collective bonus makes it possible to analyse the effects of management decisions, like a more risky investment, on the existing portfolio, the new business and shareholders.

## Conclusion

- The **concept of the collective bonus** provides a **well-defined method to** systematically **analyse the interaction of contracts** in a heterogeneous life insurance portfolio, in particular with respect to cross-subsidizing effects.
- The **concept of the collective bonus** allows to determine **a fair balancing of interests** between new business, the existing portfolio and the shareholders. In particular, our method allows to calculate a *fair* spread between different new business strategies in an “individual” and a “collective way”.
- The effects of different new business strategies on the existing portfolio, the new business and shareholders can be systematically analysed using the **concept of the collective bonus**, especially the **effects of a differentiation of surplus participation**.
- The **concept of the collective bonus** makes it possible to analyse the **effects of management decisions**, like a more risky investment, on the existing portfolio, the new business and shareholders.

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**Thank you for your attention!**

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